IMPETUS Advanced finite element analyses

Corpuscular method for airbag deployment simulations in LS-DYNA

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Preface

The idea of using a corpuscular approach to airbag modeling was born in 2004, after years of struggling with a continuum mechanical description of the gas flow. The main difficulty with a continuum formulation has not been the actual description of the gases, but rather the numerical handling of the gas-fabric interaction.

In early 2006 some initial tests with the corpuscular approach were carried out. The method is based on the kinetic molecular theory, where molecules are viewed as rigid particles obeying Newton's laws of mechanics. The only molecule-molecule and molecule-fabric interactions are perfectly elastic collisions.

The results so far are quite encouraging and the method might have the potential to become **the** method of choice in airbag deployment modeling.

This material describes the corpuscular method in LS-DYNA. It is compiled as a one-day training class, covering both theory and practical usage of the method. The class notes are accompanied by a set of simple test models that help bringing insight into possibilities and limitations of the method.

Lars Olovsson, May 2007

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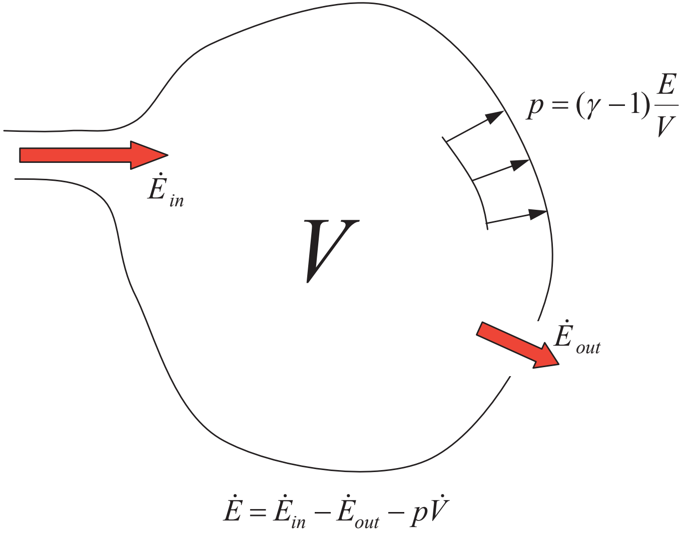
Alternative methods

Alternative methods

Control volume formulation

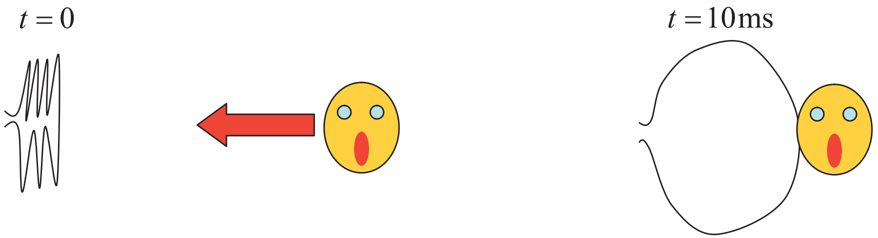
Up until today, the dominating method in airbag deployment simulations is one based on control volume assumptions, where all kinetic energy momentaneously dissipates into heat (static pressure). One normally assumes a uniform pressure distribution.



Alternative methods

Control volume formulation

Control volume assumptions are often ok if the bag deploys before interacting with the driver/passenger.



The produced results are generally less reliable in out-of-position (OOP) situations.

Alternative methods

Control volume formulation

Advantages

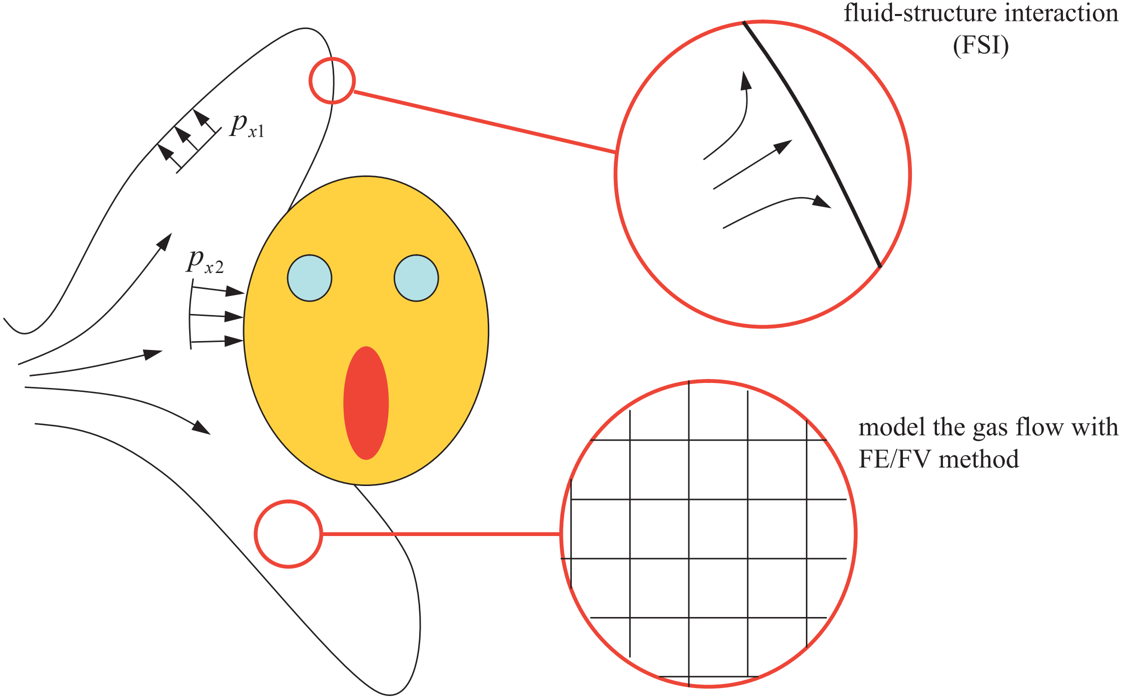
- Fast!
- Numerically robust.

Drawback

- Does not always describe reality well enough, especially not in OOP situations.

Alternative methods

Coupled Lagrangian-Eulerian formulation



fluid-structure interaction (FSI)

model the gas flow with FE/FV method

Alternative methods

Coupled Lagrangian-Eulerian formulation

Advantage

- Has a theoretical potential to produce very accurate results. The governing equations can be defined to capture significant physical effects.

Drawbacks

- Computationally expensive.
- Not a Lagrangian formulation. Numerical advection errors lead to energy dissipation and to a distorted surface of the gas body.
- Numerical difficulties in the FSI at airbag folds and at bag-to-bag contacts.
- Difficult to learn and to use.

Alternative methods

Continuum based particle methods (SPH, EFG, FPM)

Advantages

- Has a theoretical potential to produce very accurate results. The governing equations can be defined to capture significant physical effects.
- Lagrangian description of motion. Numerical advection errors are not an issue.

Drawbacks

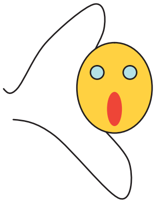
- Computationally expensive.
- Gas mixing is not trivial.
- Venting and porous leakage are not trivially handled.

Attention! I actually know too little about these methods to say anything for sure.

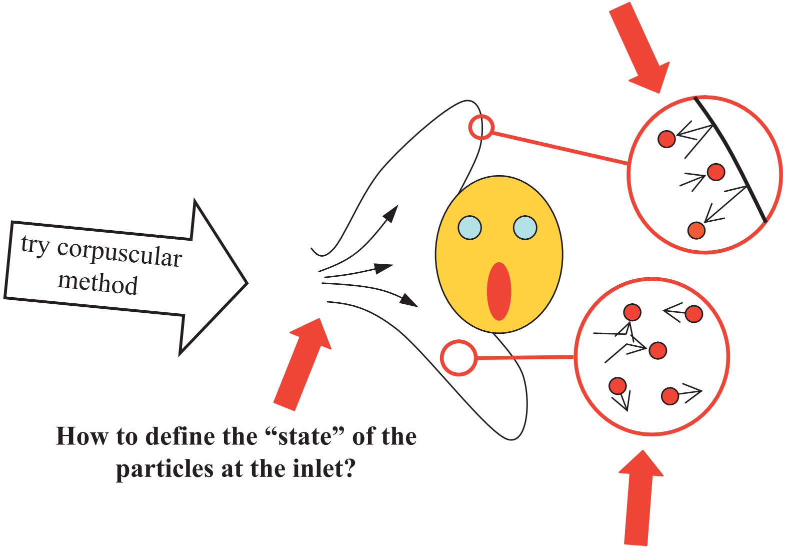
Alternative methods

Corpuscular method - questions to be answered

We want to simulate out-of-position (OOP) cases.



- Continuum based descriptions of the airbag gases lead to numerical difficulties.
- CV approach is not accurate enough.



How to obtain the correct pressure against the fabric? Definitions of porous leakage and venting?

How to define the "state" of the particles at the inlet?

How to make a cloud of rigid spheres behave like an ideal gas?

The ideal gas law

The ideal gas law

The ideal gas law can actually be derived from the kinetic molecular theory, which is the basis for the corpuscular method implemented in LS-DYNA.

However, in this context it is believed to be more natural to have the ideal gas law as a starting point. Having certain gas properties and their relationships clear in mind will make it easier to follow the derivations behind the kinetic molecular theory.

number of molecules [mol]

absolute temperature [K]

$$pV = nRT \tag{1}$$

pressure [Pa]   volume [m^3]   universal gas constant R=8.3145 [J/mol K]

As well the control volume (CV) approach as continuum formulations in airbag modeling use the ideal gas law for the constitutive description of the airbag gases. Available inflator data (temperature and mass flow rate curves) are always adapted for the ideal gas law. The corpuscular method in LS-DYNA uses the same data to define the inflator characteristics as in the CV formulation, i.e. heat capacities, mass flow rates and temperature curves. Internally, the data is automatically converted to particle properties.

The ideal gas law

Heat capacity at constant volume

We want to express the ideal gas law slightly differently than in Equation **(1)**, using heat capacities at constant volume C_v and at constant pressure C_p. Let us first have a look at the definitions of these properties.

C_v comes from the definition of the total internal energy E in an ideal gas:

$$E = \underset{\text{mass [kg]}}{m} \int_0^T \underset{\text{heat capacity at constant volume [J/kg K]}}{C_v}(\overline{T})\,\mathrm{d}\overline{T} \tag{2}$$

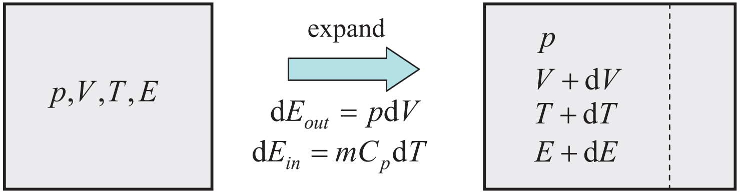
That is, C_v is defined as the energy per unit mass that is required to raise the temperature 1K while the volume is kept constant.

To some extent, C_v and C_p are both pressure and temperature dependent. However, at moderate pressures they can be assumed to only depend on the temperature.

The ideal gas law

Heat capacity at constant pressure

The heat capacity at constant pressure C_p is defined as the energy per unit mass that is required to raise the temperature 1K in a volume that is expanding during the heating process in such way that the pressure is kept constant.



p, V, T, E

expand

$$dE_{out} = p\,dV$$
$$dE_{in} = mC_p\,dT$$

p
$V + dV$
$T + dT$
$E + dE$

Internal energy increase: $dE = mC_v dT = dE_{in}\text{-}dE_{out} = mC_p dT - p\,dV$ **(3)**

Constant pressure:
$$\left.\begin{aligned} pV &= nRT \\ p(V + dV) &= nR(T + dT) \end{aligned}\right\} \Longrightarrow dT = \frac{T}{V}dV \qquad \textbf{(4)}$$

Equations **(3)** and **(4)** give: $m(C_p - C_v)\dfrac{\cancel{TdV}}{\cancel{V}} = p\,dV = \dfrac{nR\cancel{T}}{\cancel{V}}\cancel{dV}$

$$\boxed{C_p - C_v = \frac{nR}{m} = \frac{R}{M}} \qquad \textbf{(5)}$$

M [kg/mol] is the molar mass. One mole is $6.022 \cdot 10^{23}$ molecules. Note that C_p-C_v is temperature independent!

The ideal gas law

Heat capacity at constant pressure

Equations **(1)** and **(5)** can be combined into a, for many engineers, familiar expression:

$$p = \rho(C_p - C_v)T \qquad \textbf{(6)}$$

where ρ is the gas density. Assuming constant heat capacities, the specific internal energy per unit volume is: $e=\rho C_v T$ and hence:

$$e = \frac{E}{V} = \rho C_v T$$

and hence:

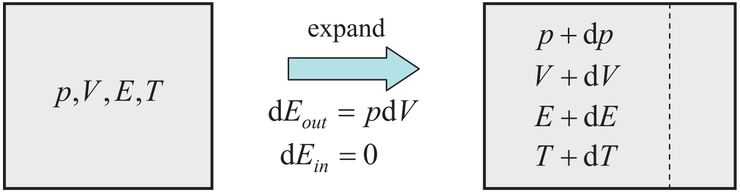
$$p = \frac{C_p - C_v}{C_v} e = (\gamma - 1)e \qquad \textbf{(7)}$$

Here $\gamma=C_p/C_v$, the ratio between heat capacities at constant pressure and constant volume. It will be shown later that $1< \gamma \leq 5/3$ (gas type dependent property).

The ideal gas law

Adiabatic expansion

During adiabatic expansion, the gas is carrying out an external work, but there is no heat exchange with the surrounding.



p, V, E, T

expand

$$\mathrm{d}E_{out} = p\,\mathrm{d}V$$

$$\mathrm{d}E_{in} = 0$$

$p + \mathrm{d}p$

$V + \mathrm{d}V$

$E + \mathrm{d}E$

$T + \mathrm{d}T$

Internal energy decrease: $\mathrm{d}E = \mathrm{d}E_{in} - \mathrm{d}E_{out} = -p\,\mathrm{d}V$

Assume constant C_v and C_p: $p = (\gamma - 1)\dfrac{E}{V}$

$$\left.\begin{array}{l} \mathrm{d}E = \mathrm{d}E_{in} - \mathrm{d}E_{out} = -p\,\mathrm{d}V \\ p = (\gamma - 1)\dfrac{E}{V} \end{array}\right\} \Longrightarrow \mathrm{d}E = -(\gamma - 1)E\frac{\mathrm{d}V}{V} \qquad \textbf{(8)}$$

Equation **(8)** gives an expression for the energy drop when expanding from volume V_0 to V_1:

$$\boxed{E_1 = E_0\left(\frac{V_0}{V_1}\right)^{\gamma - 1}} \qquad \textbf{(9)}$$

The ideal gas law

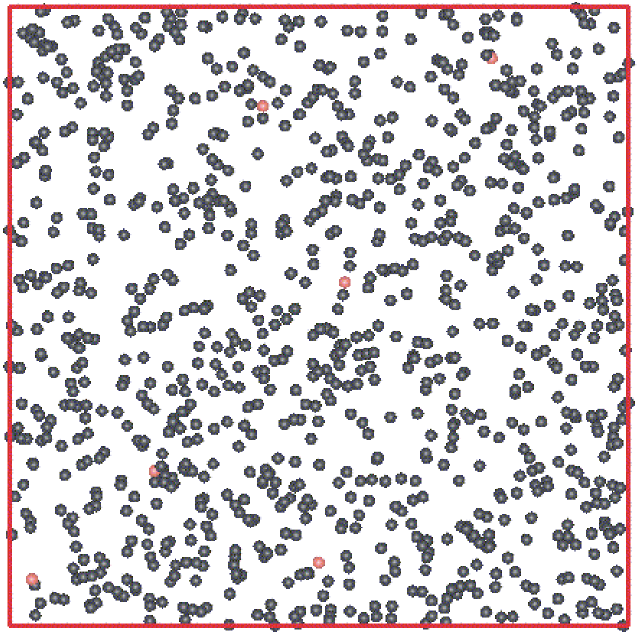
Summary

1. C_v is the heat capacity at constant volume. That is, C_v is the energy per unit mass that is required to raise the gas temperature 1K while the volume is kept constant.
2. C_p is the heat capacity at constant pressure. That is, C_p is the energy per unit mass that is required to raise the temperature 1K at constant pressure.
3. The difference C_p-C_v=R/M is temperature independent. R=8.3145 [J/kg K] is the universal gas constant and M is the molar mass of the gas.
4. Having constant heat capacities, the pressure of an ideal gas becomes $p=(\gamma-1)e$, where $\gamma = C_p/C_v$ and e is the specific internal energy per unit volume.

Kinetic molecular theory

Kinetic molecular theory

Modeling the gas as a set of rigid particles in random motion



Kinetic molecular theory

Background and assumptions

The kinetic theory is the study of gas molecules and their interaction (on a microscopic level) which leads to the ideal gas law (macroscopic relationships). The theory is based on the following assumptions:

- The average distance between the molecules is large compared to their size.
- There is a thermo-dynamical equilibrium, i.e. the molecules are in random motion.
- The molecules obey Newton's laws of motion.
- The only molecule-molecule and molecule-structure interactions are perfectly elastic collisions.

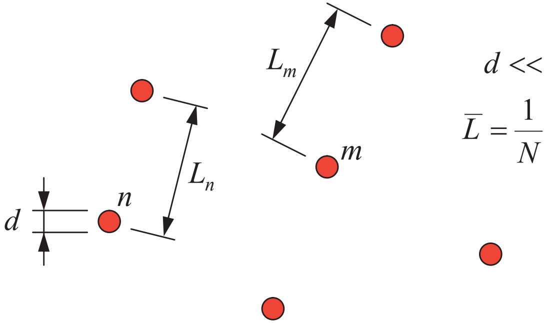
The kinetic molecular theory dates back to 1738 when Daniel Bernoulli [1], [2] proposed a theory that the air pressure against a piston is built up by discrete molecular collisions.

Having the kinetic theory as a starting point, in 1860 James Clerk Maxwell [3] derived a very elegant expression for the molecular velocity distribution at thermal equilibrium. He managed to bring more understanding to details about the molecular interaction in an ideal gas. One can, from his statistical descriptions, derive quantities such as the mean free path and frequency of collision.

Kinetic molecular theory

Background and assumptions

the average distance between the molecules is large compared to their size



$$d << \overline{L}$$

$$\overline{L} = \frac{1}{N}\sum_{i=1}^{N} L_i$$

thermal equilibrium (random motion)

$$\frac{1}{N}\sum_{i=1}^{N} v_{k,i} << v_{rms} \qquad k \in (x, y, z)$$

where

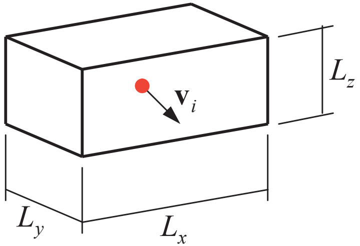
$$v_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \left|\mathbf{v}_i\right|^2}$$

Kinetic molecular theory

Pressure, one molecule

Assume one single molecule with mass m_i and velocity $\mathbf{v}_i = [v_{x,i}\ v_{y,i}\ v_{z,i}]^T$ inside a rectangular box with side lengths L_x, L_y and L_z. The frequency at which this molecule impacts the wall at $x{=}L_x$ becomes:

$$f = \frac{|v_{x,i}|}{2L_x} \qquad \textbf{(10)}$$



The impulse transferred to the wall each impact is:

$$j_{x,i} = 2m_i|v_{x,i}| \qquad \textbf{(11)}$$

Over time the total impulse transferred to the wall becomes:

$$J_{x,i} = j_{x,i} f_{x,i} t = \frac{m_i|v_{x,i}|^2 t}{L_x} \qquad \textbf{(12)}$$

Hence, the average pressure against the wall is:

$$p_{x,i} = \frac{J_{x,i}}{At} = \frac{m_i|v_{x,i}|^2}{L_x L_y L_z} = \frac{m_i|v_{x,i}|^2}{\boxed{V}} \qquad \textbf{(13)}$$

volume of box

Kinetic molecular theory

Pressure, many molecules

Having N molecules the pressure can be summed up as:

$$p_x = \sum_{i=1}^{N} p_{x,i} = \frac{1}{V} \sum_{i=1}^{N} m_i |v_{x,i}|^2 \qquad \textbf{(14)}$$

At thermal equilibrium the kinetic energy is evenly distributed to the different Cartesian directions. Hence:

$$\sum_{i=1}^{N} m_i |v_{x,i}|^2 = \sum_{i=1}^{N} m_i |v_{y,i}|^2 = \sum_{i=1}^{N} m_i |v_{z,i}|^2 = \frac{1}{3} \sum_{i=1}^{N} m_i |v_i|^2 = \frac{2}{3} W_k \qquad \textbf{(15)}$$

where W_k is the total translational kinetic energy of all molecules. Hence:

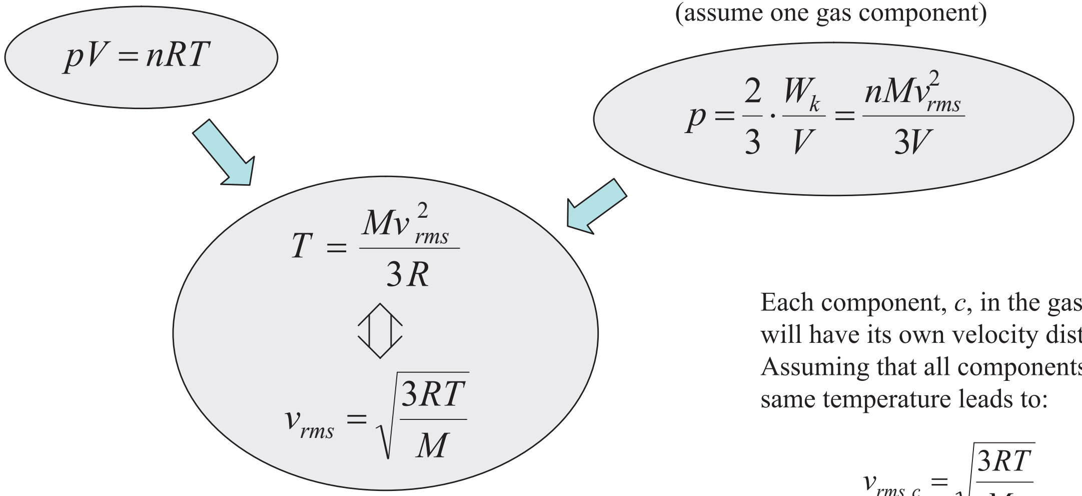
$$p_x = p_y = p_z = \boxed{p = \frac{2W_k}{3V} = \frac{2}{3} w_k} \qquad \textbf{(16)}$$

w_k is the specific translational kinetic energy per unit volume.

Kinetic molecular theory

Molecular velocity and temperature

We are now ready for a definition of temperature that will act as a link between the ideal gas law and the kinetic molecular theory:



Ideal gas law Equation **(1)**

$$pV = nRT$$

Kinetic molecular theory, Equation **(16)**
(assume one gas component)

$$p = \frac{2}{3} \cdot \frac{W_k}{V} = \frac{nMv_{rms}^2}{3V}$$

$$T = \frac{Mv_{rms}^2}{3R} \Updownarrow v_{rms} = \sqrt{\frac{3RT}{M}}$$

Each component, *c*, in the gas mixture will have its own velocity distribution. Assuming that all components have the same temperature leads to:

$$v_{rms,c} = \sqrt{\frac{3RT}{M_c}} \qquad \textbf{(17)}$$

Kinetic molecular theory

Pressure and internal energy

Macroscopically the translational kinetic energy per unit volume w_k is a fraction $\xi(T)$ of the specific internal energy e of the gas.

$$w_k = \xi(T)e \Rightarrow \left|\text{Equation } \mathbf{(16)}\right| \Rightarrow p = \frac{2}{3} w_k = \frac{2}{3}\xi(T)e \qquad \mathbf{(18)}$$

Actually, this fraction is a direct function of the heat capacities (at thermal equilibrium). Equations **(6)** and **(18)** give:

$$\left.\begin{array}{r} p = \frac{2}{3}\xi(T)e \\ p = \rho(C_p - C_v)T \end{array}\right\} \Rightarrow \xi(T) = \frac{3\rho(C_p - C_v)T}{2e} \qquad \mathbf{(19)}$$

Assuming temperature independent heat capacities such that $e=\rho C_v T$ this relation boils down to:

$$\xi = \frac{3}{2} \cdot \frac{C_p - C_v}{C_v} = \frac{3}{2}(\gamma - 1) \qquad \mathbf{(20)}$$

Kinetic molecular theory

Ratio between heat capacities

Mono-atomic gases (e.g. He and Ar) store virtually no energy as vibrations or spin. Hence, $w_k = e$ and $\xi = 1$.

$$\xi = 1 \quad \Rightarrow \quad \gamma = 5/3 \approx 1.6667$$

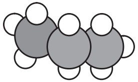
$\gamma = 5/3$ is a theoretical upper limit. It is not possible to store more than 100% of the internal energy as translational kinetic energy.

Di-atomic gases (e.g. N_2 and O_2) store some energy as spin and, at elevated temperatures, as molecular vibrations. At moderate temperatures roughly 60% is translational kinetic energy and 40% is spin. That is $\xi = 3/5$.

$$\xi = 3/5 \quad \Rightarrow \quad \gamma = 7/5 = 1.4$$

The more complex molecules, the more energy is stored as vibration and spin. However, the fraction of translational kinetic energy can not reach or drop below zero. Hence, $\gamma > 1$ for all gases. The upper and lower limits give us:

$$\boxed{1 < \gamma \leq 5/3} \qquad \textbf{(21)}$$

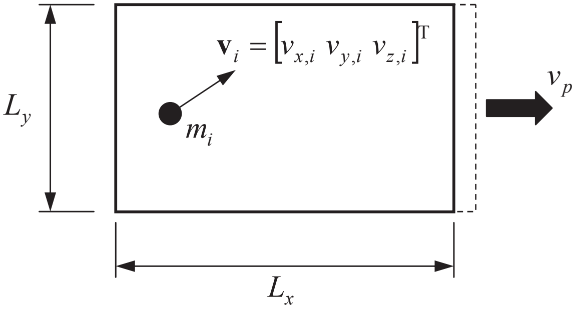


Propane $\gamma \approx 1.13$ (at 300K)

Kinetic molecular theory

Adiabatic expansion

Assume a molecule inside a slowly expanding box with side lengths L_x, L_y and L_z according to the figure below.



The particle will lose some energy when impacting the moving wall. It can be shown, using conservation of momentum and energy, that the particle velocity in x-direction after impact becomes $2v_p$-$v_{x,i}$.

$$\Delta E_i = \frac{1}{2} m_i \underbrace{(v_{x,i} - 2v_p)}_{\text{velocity after elastic impact}}{}^2 - \frac{1}{2} m_i \underset{\substack{\uparrow \\ \text{velocity before impact}}}{v_{x,i}^2} \approx \left| v_p << v_{x,i} \right| \approx -2 m_i v_{x,i} v_p \tag{22}$$

Kinetic molecular theory

Adiabatic expansion

Combining this with the impact frequency in Equation **(10)** gives a rate of dropping energy.

$$\dot{E}_i = f\,\Delta E_i = -\frac{m_i v_{x,i}^2 v_p}{L_x} = -m_i v_{x,i}^2 \frac{\dot{V}}{V} \tag{23}$$

Having many particles inside the box, the total rate of dropping energy due to impacts against the moving wall becomes:

$$\dot{E} = \sum_{i=1}^{N} \dot{E}_i = -\sum_{i=1}^{N} m_i v_{x,i}^2 \frac{\dot{V}}{V} = \left|\text{Equation } \mathbf{(15)}\right| = -\frac{2W_k}{3} \cdot \frac{\dot{V}}{V} \tag{24}$$

We know that W_k is a fraction ξ of the total internal energy E in the gas. Hence:

$$\dot{E} = -\frac{2\xi E}{3} \cdot \frac{\dot{V}}{V} \Rightarrow \left|\begin{matrix}\text{assume temperature} \\ \text{independent } \xi\end{matrix}\right| \Rightarrow E_1 = E_0 \left(\frac{V_0}{V_1}\right)^{\frac{2\xi}{3}} = E_0 \left(\frac{V_0}{V_1}\right)^{\gamma - 1} \tag{25}$$

Hence, the kinetic theory predicts the same energy drop as when working with the ideal gas law (see Equation **(9)**).

Kinetic molecular theory

Maxwell-Boltzmann distribution of velocities

The Maxwell-Boltzmann distribution of molecular velocities in an ideal gas is based on one simple assumption: The velocity distribution in different orthogonal directions are uncoupled. That is, the probability of having a specific velocity in *x*-direction is the same, no matter which velocity the molecule has in *y*- and *z*-directions. From this single assumption one can show that the velocity distribution at thermal equilibrium becomes:

molar mass

velocity distribution function

$$f(|\mathbf{v}|) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2}|\mathbf{v}|^2\exp\left(\frac{-M|\mathbf{v}|^2}{2RT}\right) \tag{26}$$

universal gas constant   temperature

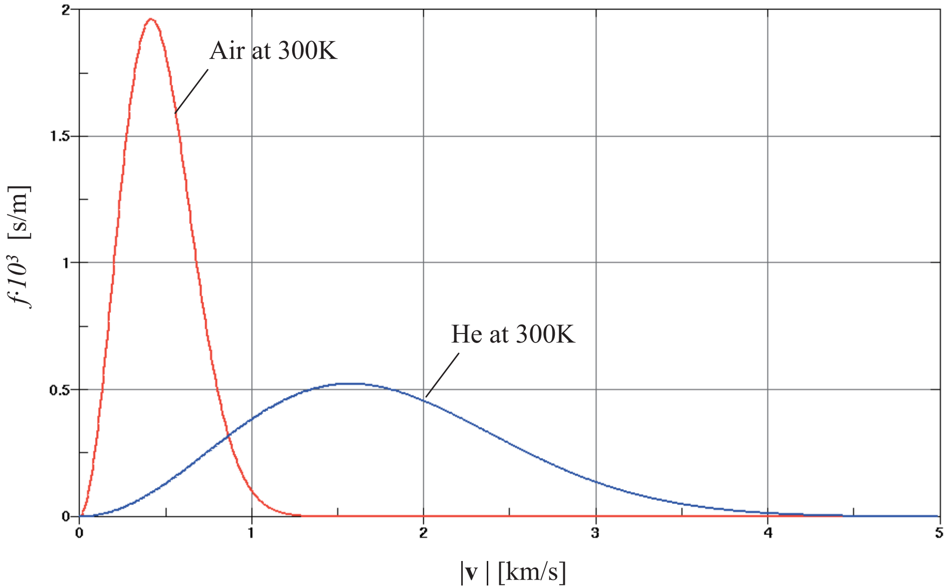
The velocity distribution function is also valid for gas mixtures, where different components may have different molar masses. This is displayed in one of the numerical examples (page 62).

In LS-DYNA the velocity distribution function is needed for an accurate translation of macroscopic properties, such as heat capacities, to particle data.

Kinetic molecular theory

Maxwell-Boltzmann distribution of velocities

Air and helium at 300K. Helium has a lower molar mass and, consequently, larger molecular velocities.



Kinetic molecular theory

Frequency of collision with a fabric segment

The frequency of collision with a fabric segment can be derived directly from Maxwell-Boltzmann's velocity distribution. Assuming only one gas component:

fabric segment area

number of molecules

root-mean-square velocity
of molecules

$$f_c = \frac{N A_{seg} v_{rms}}{\sqrt{6\pi V}} \qquad \textbf{(27)}$$

bag volume

This equation is very useful when estimating the level of noise in the particle-fabric contact pressure. Keeping the noise level constant, there is a linear relationship between the bag volume and the required number of particles.

Kinetic molecular theory

Summary

1. The specific internal energy in an ideal gas can be divided into translational kinetic energy, vibrations and spin. It is the translational kinetic energy that produces pressure.
2. The kinetic molecular theory and the ideal gas law predict the same pressure at thermal equilibrium (given the relationship between molecular velocity and temperature on page 25).
3. The kinetic molecular theory matches the ideal gas law in adiabatic expansion.
4. Since the pressure is a function of the specific translational kinetic energy only, a few large molecules with total mass m_{tot} will produce the same pressure as a many small molecules with the same total mass, as long as their root mean square velocities v_{rms} are the same.

   This is of fundamental importance for the corpuscular method in LS-DYNA.

Corpuscular method in LS-DYNA

Corpuscular method in LS-DYNA

Reduce system from many molecules to a “few” particles

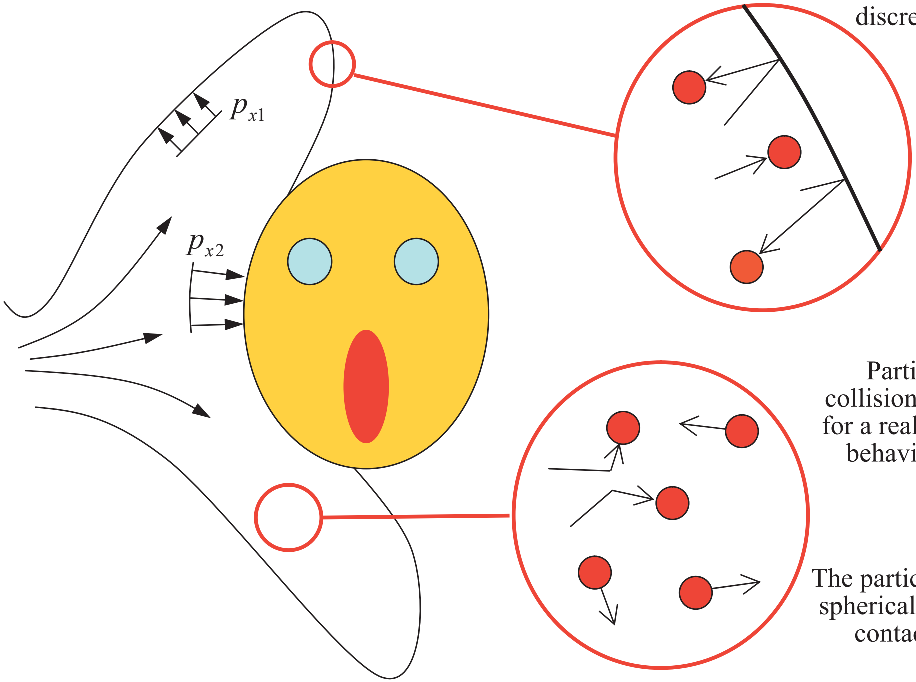
One can not possibly model every single molecule inside the bag. That is why one normally reverts to a continuum treatment of the gas and to numerical methods such as FEM, SPH or EFG. However, as mentioned before, the numerical difficulties associated with continuum formulations are not trivial.

Instead of a continuum formulation, let us try to replace the molecules inside the bag with a moderate number of particles. Letting a couple of particles bounce around in the bag should give us a fairly simple gas-fabric contact treatment. For sure, contact will be simpler than with a continuum formulation, but one question remains to be answered: **How well can a few particles describe the behavior of the gas?**

For airbag simulations, it is of utmost importance to manage predicting both a static gas pressure and the evolution of pressure as the gas expands during the deployment process.

Corpuscular method in LS-DYNA

Reduce system from many molecules to a “few” particles



Pressure is built up by discrete particle-fabric impacts.

Particle-particle collisions are necessary for a realistic dynamical behavior of the gas.

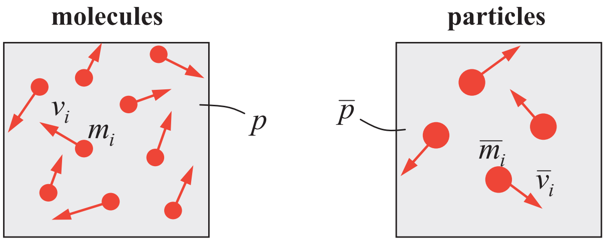
The particles are assumed spherical for an efficient contact treatment.

Corpuscular method in LS-DYNA

Reduce system from many molecules to a "few" particles

Static pressure

We know that the static pressure is a direct function of the translational kinetic energy in the gas. Hence, one can match the expected pressure with a few particles, as long as their total translational kinetic energy is correct.



$$W_k = \frac{1}{2}\sum_{i=1}^{N_m} m_i\, v_i^2 \qquad \overline{W}_k = \frac{1}{2}\sum_{i=1}^{N_p} \overline{m}_i\, \overline{v}_i^2$$

$$\overline{W}_k = W_k \quad \Rightarrow \quad \overline{p} = p$$

Corpuscular method in LS-DYNA

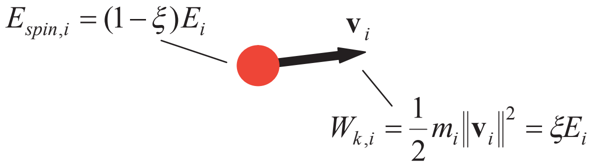
Reduce system from many molecules to a "few" particles

Adiabatic expansion

We have seen in Equation **(25)** that the energy drop at adiabatic expansion is a function of the ratio ξ between translational kinetic energy and total internal energy in the gas (some energy is stored as molecular vibration and spin). The ratio is a direct function of the heat capacities.

$$\dot{E} = -\frac{2\xi E}{3} \cdot \frac{\dot{V}}{V}$$

One must make sure that the particles together carry the same amount of spin/vibration energy as the real gas. This can be done by assigning a lumped amount of spin/vibration energy to each particle. This additional energy should be chosen such that the translational kinetic energy becomes exactly the fraction ξ of the total energy.



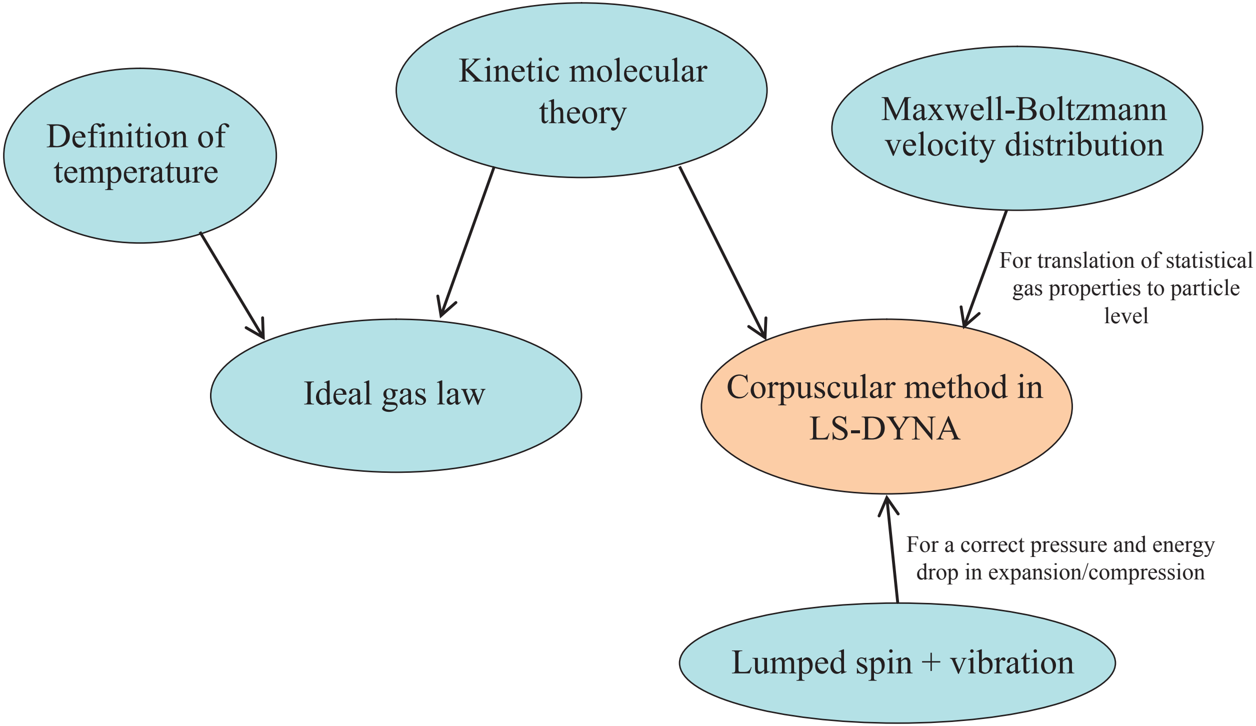
$$E_{spin,i} = (1-\xi)E_i$$

$$\mathbf{v}_i$$

$$W_{k,i} = \frac{1}{2} m_i \left\| \mathbf{v}_i \right\|^2 = \xi E_i$$

Corpuscular method in LS-DYNA

Reduce system from many molecules to a “few” particles



Definition of temperature

Kinetic molecular theory

Maxwell-Boltzmann velocity distribution

Ideal gas law

For translation of statistical gas properties to particle level

Corpuscular method in LS-DYNA

For a correct pressure and energy drop in expansion/compression

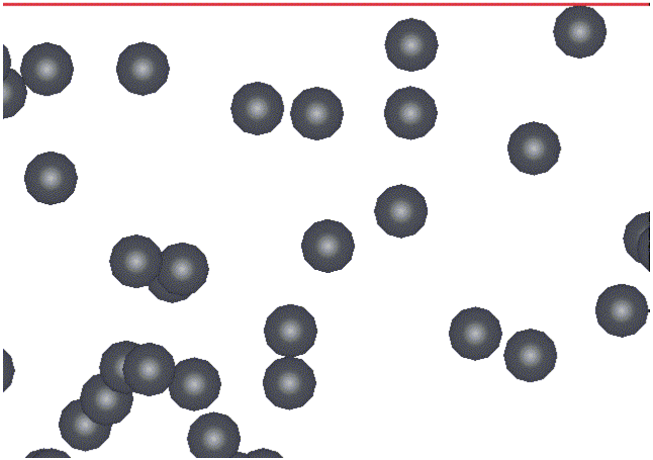
Lumped spin + vibration

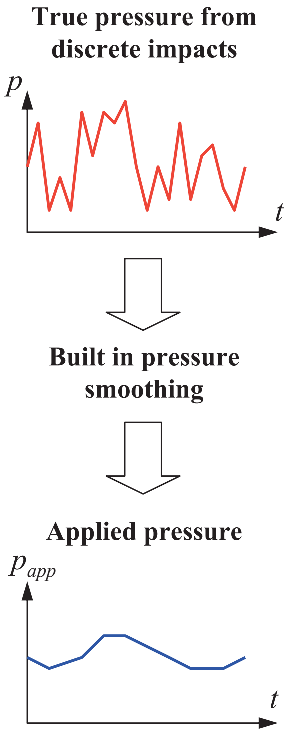
Corpuscular method in LS-DYNA

Noise

We can generally not afford enough many particles to produce a locally smooth pressure response. A built in pressure smoothing limits the noise level, on the expense of the local momentum balance.

Note that the particle radius is neglected in the particle-fabric contact. This is important for a realistic leakage through small vent holes.





True pressure from discrete impacts

Built in pressure smoothing

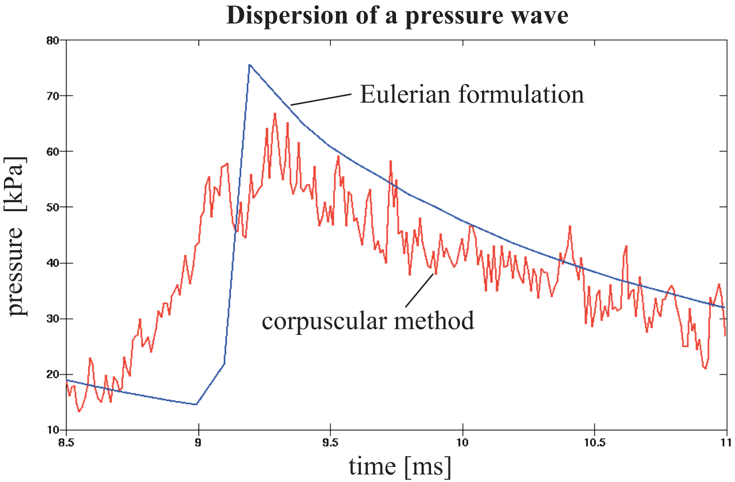
Applied pressure

Corpuscular method in LS-DYNA

Dispersion

Going from many molecules to a few particles drastically increases the mean free path. As a consequence, the diffusion is massively over estimated. This has a negative impact on the ability to resolve pressure waves. Waves tend to disperse very quickly.

The molecules in air at atmospheric pressure and at room temperature have a mean free path around 70nm. Replacing the molecules with 10,000 particles per liter gives (with the current implementation in LS-DYNA) a mean free path of 5.0mm. That is, roughly 70,000 times longer than in reality!



Corpuscular method in LS-DYNA

Summary

Advantages

- Simple and numerically very robust.
- Lagrangian description of motion.
- Straight-forward treatment of venting, porous leakage and gas mixing.

Drawbacks

- Noisy
- Diffusion is heavily exaggerated. Pressure waves are quickly smeared out.

Corpuscular method in LS-DYNA

Summary

1. The corpuscular method in LS-DYNA is based on the kinetic molecular theory. However, each particle is defined to represent many molecules.
2. The particles are given a spherical shape for an efficient contact treatment.
3. For each particle there is a balance between translational energy and spin+vibrations. This balance is determined directly from the heat capacities (or from ξ).
4. Letting each particle represent many molecules leads to dispersion and to a noisy particle-fabric contact pressure. The noise is reduced by smearing out the applied pressure in time.
5. The absence of field equations makes the method numerically simple and robust.

*AIRBAG_PARTICLE

*AIRBAG_PARTICLE

Keyword structure

There is only one keyword command associated with the corpuscular method in LS-DYNA and it is called ***AIRBAG_PARTICLE**. The input structure is set up to make a conversion from ***AIRBAG_HYBRID** as simple as possible. ***AIRBAG_PARTICLE** defines:

1. Airbag parts (external, internal and vent holes).
2. External air properties.
3. Inflator gas properties, mass flow rates and inlet temperature curves.
4. Vent hole characteristics (functions of pressure and time).
5. Inflator location, geometry and nozzle directions.

Note that porous properties of the airbag fabric are defined in ***MAT_FABRIC**.

*AIRBAG_PARTICLE

Keyword structure

***AIRBAG_PARTICLE**

SID1	STYPE1	SID2	STYPE2	BLOCK	HCONV		
NP	UNIT	VISFLG	TATM	PATM	NVENT	TEND	TSW
IAIR	NGAS	NORIF	NID1	NID2	NID3		

NVENT cards

SID3	STYPE3	C23	LCTC23	LCPC23

Optional card if IAIR=1

PAIR	TAIR	XMAIR	AAIR	BAIR	CAIR

NGAS cards

LCMi	LCTi	XMi	Ai	Bi	Ci	INFGi

NORIF cards

NIDi	ANi	VDi	CAi	INFOi

*AIRBAG_PARTICLE

Keyword structure

CARD 1

SID1	- Set defining the complete bag
STYPE1	- Set type Eq.0: Part Eq.1: Part set
SID2	- Set defining the internal parts of the bag
STYPE2	- Set type Eq.0: Part Eq.1: Part set
BLOCK	- Blocking Eq.0: Off Eq.1: On
HCONV	- Future parameter for convective heat transfer

*AIRBAG_PARTICLE

Keyword structure

CARD 2

NP - Number of particles
UNIT - Unit system
    Eq.0: kg-mm-ms-K
    Eq.1: SI-units
    Eq.2: ton-mm-s
VISFLG - Visible particles
    Eq.0: No
    Eq.1: Yes
TATM - Atmospheric temperature (default 293K)
PATM - Atmospheric pressure (default 101.3kPa)
NVENT - Number of vent hole definitions
TEND - Time when all particles have entered the bag (default 1.0e10)
TSW - Time for switch to control volume formulation (default 1.0e10)

*AIRBAG_PARTICLE

Keyword structure

CARD 3

IAIR	- Initial air inside the bag considered Eq.0: no Eq.1: yes
NGAS	- Number of gas components
NORIF	- Number of orifices
NID1-NID3	- Three nodes defining a moving coordinate system for the direction of flow through the gas inlet nozzles

*AIRBAG_PARTICLE

Keyword structure

NVENT cards

SID3	- Set defining vent holes
STYPE3	- Set type Eq.0: part Eq.1: part set
C23	- Vent hole coefficient (parameter for Wang-Nefske leakage) (default 1.0)
LCTC23	- Load curve defining vent hole coefficient as a function of time
LCPC23	- Load curve defining vent hole coefficient as a function of pressure

If IAIR=1

PAIR	- Initial pressure inside bag (default PAIR=PATM)
TAIR	- Initial temperature inside bag (default TAIR=TATM)
XMAIR	- Molar mass of air initially inside bag
AAIR-CAIR	- Constant, linear and quadratic heat capacities at constant pressure [J/mol K]

*AIRBAG_PARTICLE

Keyword structure

NGAS cards

LCM*i*	- Mass flow rate curve for component *i*
LCT*i*	- Temperature load curve for component *i*
XM*i*	- Molar mass of component *i*
A*i*-C*i*	- Constant, linear and quadratic heat capacities at constant pressure [J/mol K]
INFG*i*	- Inflator ID that this gas component belongs to

NORIF cards

NID*i*	- Node ID defining location of nozzle *i*
AN*i*	- Area of nozzle *i*
VD*i*	- ID of vector defining initial direction of gas inflow at nozzle *i*
CA*i*	- Cone angle in radians (jet angle, default 30°)
INFO*i*	- Inflator ID that orifice *i* belongs to

*AIRBAG_PARTICLE

Temperature curves

The inlet temperature curves describe the static temperature. The stagnation temperature is higher. Assuming constant heat capacities:

Inflow energy rate:

$$\dot{E}_{in} = \overbrace{\dot{m}_{in} C_v T_{stat}}^{\text{internal energy of gas}} + \overbrace{p_{in} v_{in} A_{in}}^{pV\text{-work}} =$$

$$\dot{m}_{in} C_v T_{stat} + \rho_{in}(C_p - C_v) T_{stat} v_{in} A_{in} = \dot{m}_{in} C_p T_{stat}$$

$$\dot{E}_{in} = \dot{m}_{in} C_v T_{stag}$$

$$\boxed{T_{stag} = \gamma T_{stat}} \qquad \textbf{(28)}$$

The particle velocity at the inlet is a direct function of the stagnation temperature:

$$v_{rms,inlet} = \sqrt{\frac{3RT_{stag}}{M}} \qquad \textbf{(29)}$$

*AIRBAG_PARTICLE

Heat capacities

The heat capacities in the input deck have the units [J/mol K]. We have used the units [J/kg K] for all our previous derivations.

Heat capacities per mole [J/mol K]:

$$\left.\begin{aligned}\overline{C}_p &= A + BT + CT^2\\ \overline{C}_v &= \overline{C}_p - R\end{aligned}\right\}\text{ Used in }\textbf{*AIRBAG\_PARTICLE}$$

Heat capacities per unit mass [J/kg K]:

$$C_p = \frac{\overline{C}_p}{M}$$

$$C_v = C_p - \frac{R}{M}$$

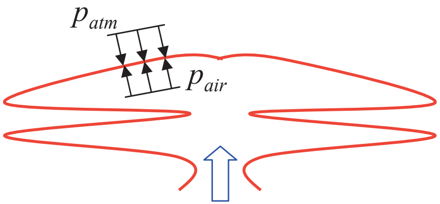
*AIRBAG_PARTICLE

Initial air

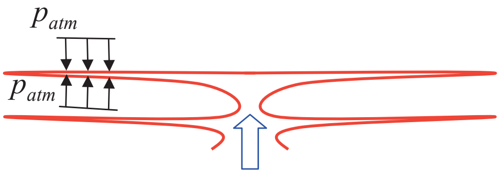
Air inside the folded bag at time zero (IAIR=1) is not modeled with particles. Instead a CV formulation is used. The air temperature is assumed equivalent to the average particle temperature. Energy is transferred between particles and air (both ways) to ensure this balance.

Initial air is important to balance the external atmospheric pressure. 1 atm from outside and no pressure inside may cause contact problems and difficulties opening the bag.

Initial air considered
pressure balance at *t=0*



No initial air
layers get squeezed together

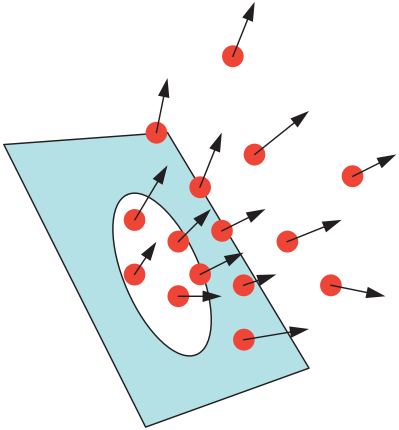


*AIRBAG_PARTICLE

Venting

We simply let the gas dynamics take care of the venting. A pressure gradient will establish near the vent hole and particles leak out. However, due to the relatively poor performance of the particle method, the venting is generally slightly under estimated. Increasing the number of particles improves the results.

C23 can be used to decrease the venting. For example, C23=0.6 will force 40% of the particles reaching the vent hole to bounce back. **Venting coefficients above unity C23>1 will have no effect on the solution**. We can simply not let through more than 100% of the particles…



Venting leakage according to Wang and Nefske
(used in CV calculations)

$$\dot{m}_{vent} = C_{23} A_{vent} \cdot \frac{pQ^{1/\gamma}}{R\sqrt{T}} \sqrt{2 \cdot \frac{\gamma}{\gamma - 1} \cdot \frac{R}{M} \cdot (1 - Q^{\frac{\gamma - 1}{\gamma}})}$$

where

$$Q = \max\left[\frac{p_{atm}}{p}, \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}\right]$$

*AIRBAG_PARTICLE

Fabric porosity

Porous properties of the fabric are defined in ***MAT_FABRIC**. The corpuscular method is adapted to match the control volume formulation OPT=7 and 8 in ***AIRBAG_WANG_ NEFSKE**

***MAT_FABRIC**

MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
GAB	GBC	GCA	CSE	EL	PRL	LRATIO	DAMP
AOPT	**FLC**	**FAC**	ELA	LNRC	FORM	FVOPT	TSRFAC
			A1	A2	A3		
V1	V2	V3	D1	D2	D3	BETA	
LCA	LCB	LCUA	LCUB	LCUAB			

FLC Fabric area coefficient

LT.0.0: |FAC| is the ID of a load curve scaling the effective fabric area in time

GT.0.0: Fabric area scale factor

FAC Fabric leakage velocity

LT.0.0: |FLC| is the ID of a load curve defining leakage velocity as function of pressure: $v_{leak} = f(p_c)$ where p_c is the pressure of gas component c.

GT.0.0: v_{leak} = FLC

*AIRBAG_PARTICLE

Fabric porosity

Porous leakage rate of gas component *c* that the corpuscular method tries to match:

area coefficient from ***MAT_FABRIC**
(fixed value or load curve)

leakage velocity from ***MAT_FABRIC**
(fixed value or load curve)

$$\dot{m}_{porous,c} = \mathrm{FLC} \cdot \rho_c v_{leak} A_{seg}$$

mass flow rate of
component *c*

local density of gas
component *c*

area of fabric
segment

Output and post-processing

abstat_cpm (in binout)

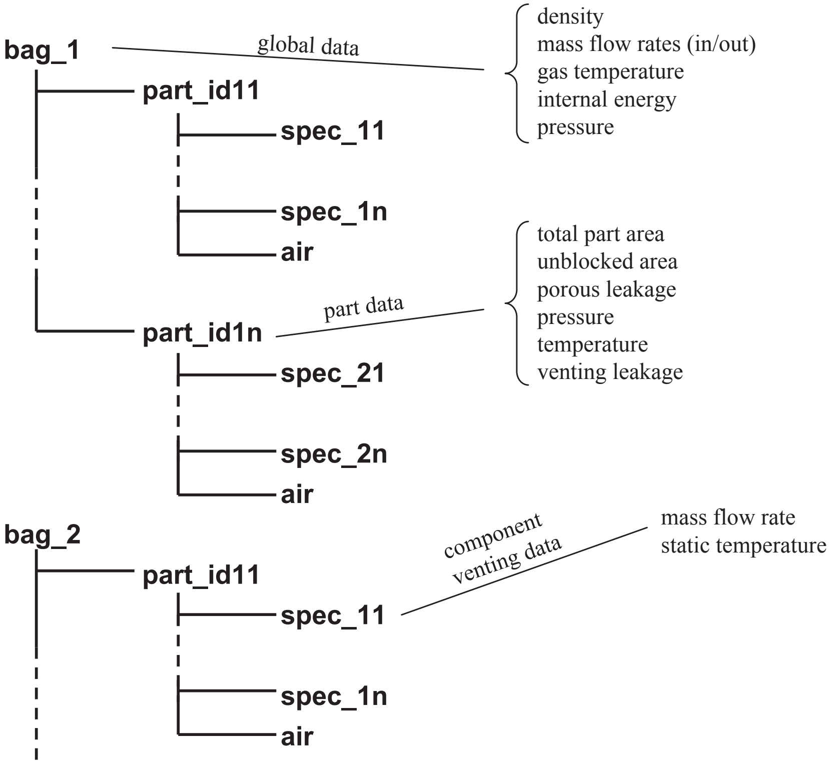
Global data, such as average pressure, temperature and total mass flow rates are written to the airbag statistics file abstat. If asking for binary output in ***DATABASE_ABSTAT**, a more detailed set of data is written to the branch abstat_cpm in binout.

***DATABASE_ABSTAT**
DT, BINARY

DT	- Output interval
BINARY	- Flag for binary output Eq.1: ASCII (default in SMP) Eq.2: Binary output Eq.3: ASCII and binary output

Output and post processing

abstat_cpm (in binout)

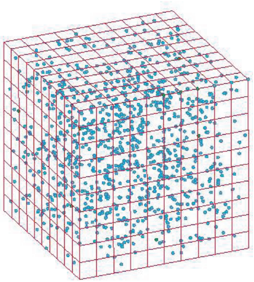


- **bag_1** — global data: density, mass flow rates (in/out), gas temperature, internal energy, pressure
  - **part_id11**
    - **spec_11**
    - **spec_1n**
    - **air**
  - **part_id1n** — part data: total part area, unblocked area, porous leakage, pressure, temperature, venting leakage
    - **spec_21**
    - **spec_2n**
    - **air**
- **bag_2**
  - **part_id11**
    - **spec_11** — component venting data: mass flow rate, static temperature
    - **spec_1n**
    - **air**

Numerical examples

Numerical examples

Thermal equilibrium, helium and air



A 1 liter box gets filled up with 0.1g air and 0.1g helium at a temperature of 300K.

***AIRBAG_PARTICLE**	
1, 0	Box is part ID 1
10000, 1, 1	10k particles, SI-units and visible particles
0, 2, 1	No initial air, 2 gas components and 1 orifice
1, 2, 0.028, 28.0	Air: mass flow rate curve ID=1, Temperature curve ID=2
1, 3, 0.003, 20.79	He: mass flow rate curve ID=1, Temperature curve ID=3
100000, 1.0	Node ID 100000 represents the orifice

$Mass flow rate curve
***DEFINE_CURVE**
1
0.0, 0.1
0.9e-3, 0.1
1.1e-3, 0.0
1.0, 0.0

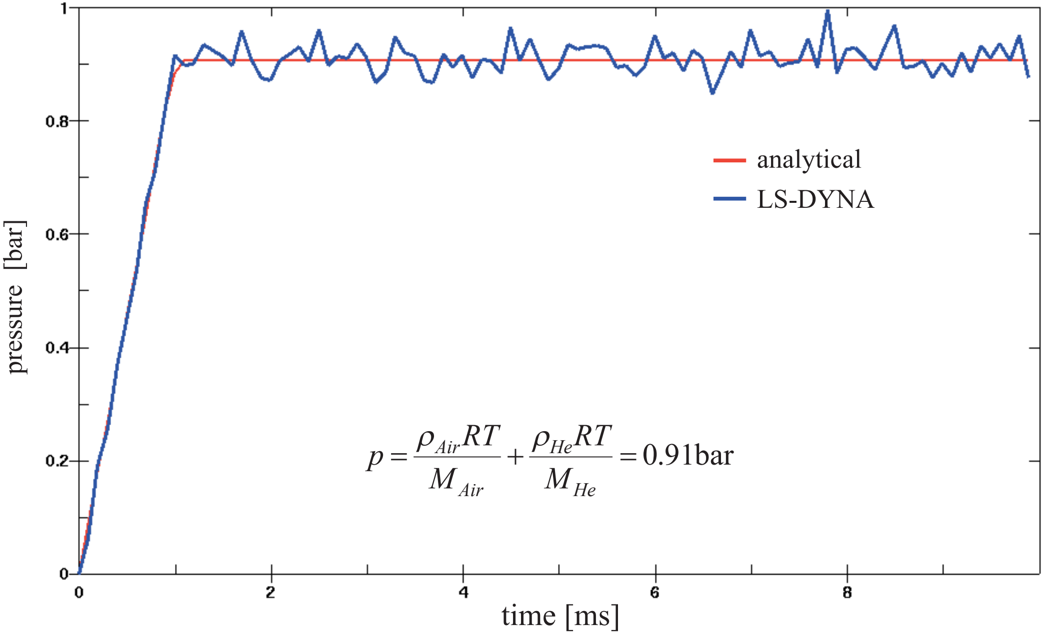
$Static air temp = 300/ γ_{Air}
***DEFINE_CURVE**
2
0.0, 210.9
1.0, 210.9

$Static He temp = 300/ γ_{He}
***DEFINE_CURVE**
3
0.0, 180.0
1.0, 180.0

Numerical examples

Thermal equilibrium, helium and air

Pressure versus time using 10,000 particles.

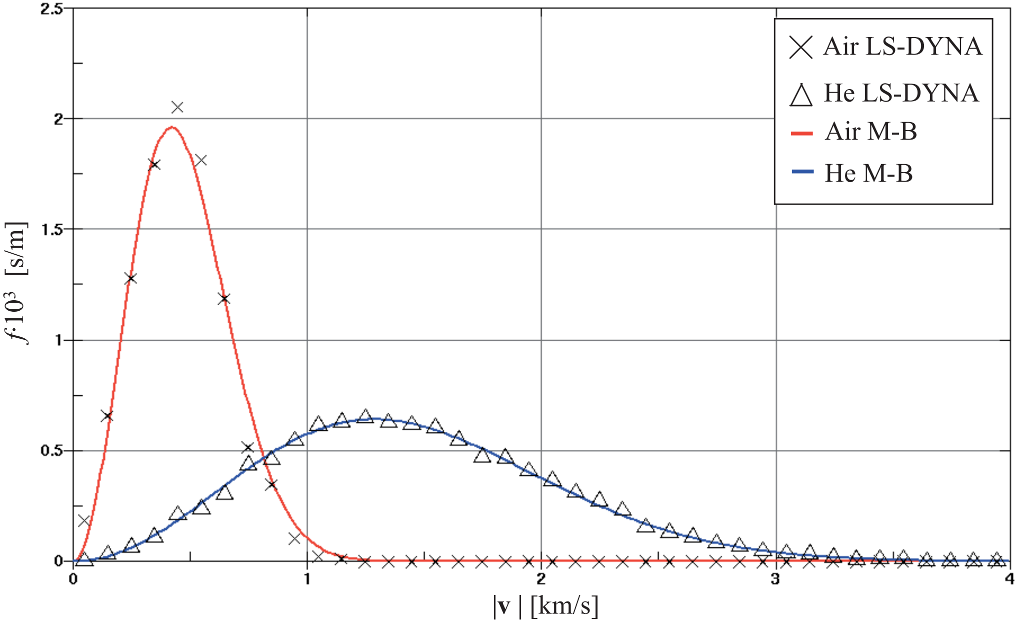


$$p = \frac{\rho_{Air} RT}{M_{Air}} + \frac{\rho_{He} RT}{M_{He}} = 0.91\text{bar}$$

Numerical examples

Thermal equilibrium, helium and air

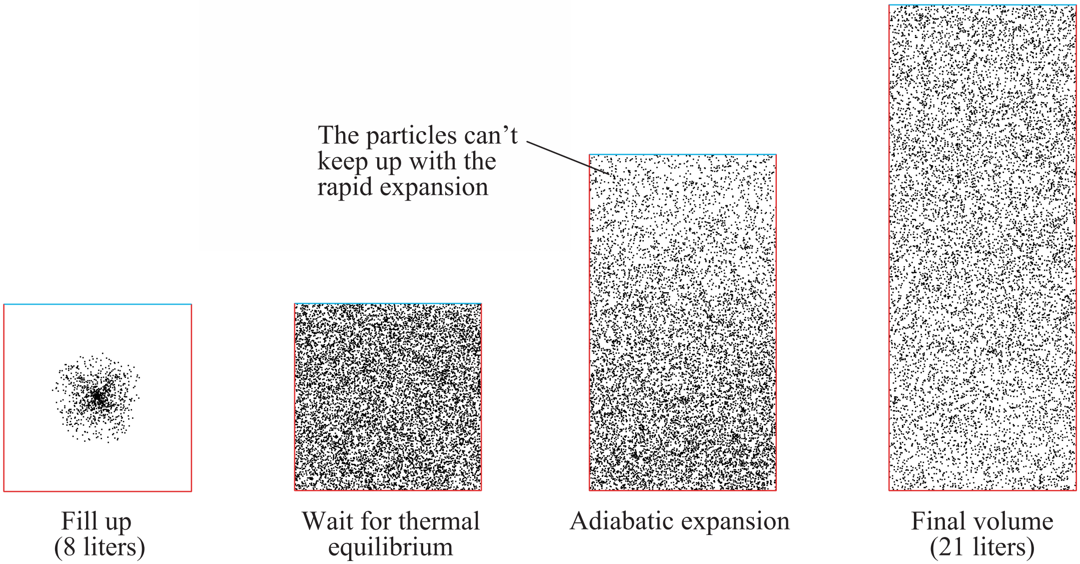
Velocity distribution at 10ms compared to analytical expression (Maxwell-Boltzmann).



Numerical examples

Quick adiabatic expansion

An 8 liter box gets filled up with 10g air at 300K. Subsequently the box is expanded to 21 liter in just 1ms (prescribed motion). The maximum velocity of the moving wall is 400m/s.



Numerical examples

Quick adiabatic expansion

The particle keyword is quite simple.

```
*AIRBAG_PARTICLE
1, 1                    Box is part set ID 1
10000, 1, 1             10k particles, SI-units and visible particles
0, 1, 1                 No initial air, 1 gas component and 1 orifice
1, 2, 0.028, 28.0       Air: mass flow rate curve ID=1, Temperature curve ID=2
12345, 1.0              Node ID 12345 represents the orifice
```

```
$Mass flow rate curve
*DEFINE_CURVE
1
0.0,        10.0
0.9e-3,     10.0
1.1e-3,     10.0
1.0,        0.0
```

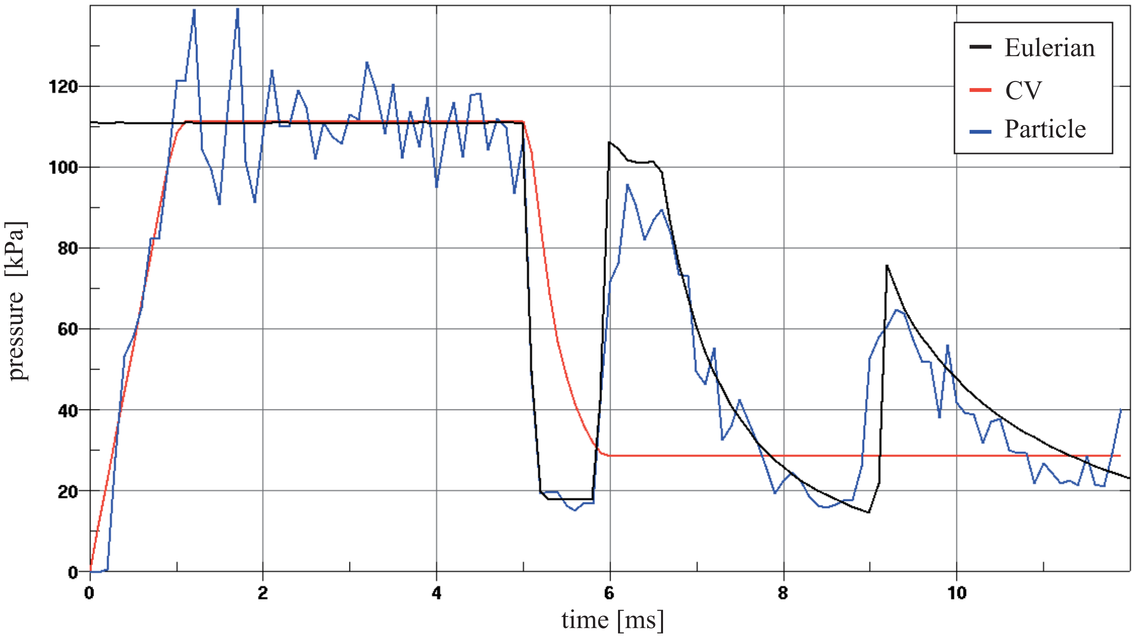
```
$Static air temp = 300/ γ_Air
*DEFINE_CURVE
2
0.0,        210.9
1.0,        210.9
```

```
$Piston motion
*DEFINE_CURVE
3
0.0,        0.0
5.0e-3,     0.0
5.2e-3,     400.0
5.8e-3,     400.0
6.0e-3,     0.0
1.0,        0.0
```

Numerical examples

Quick adiabatic expansion

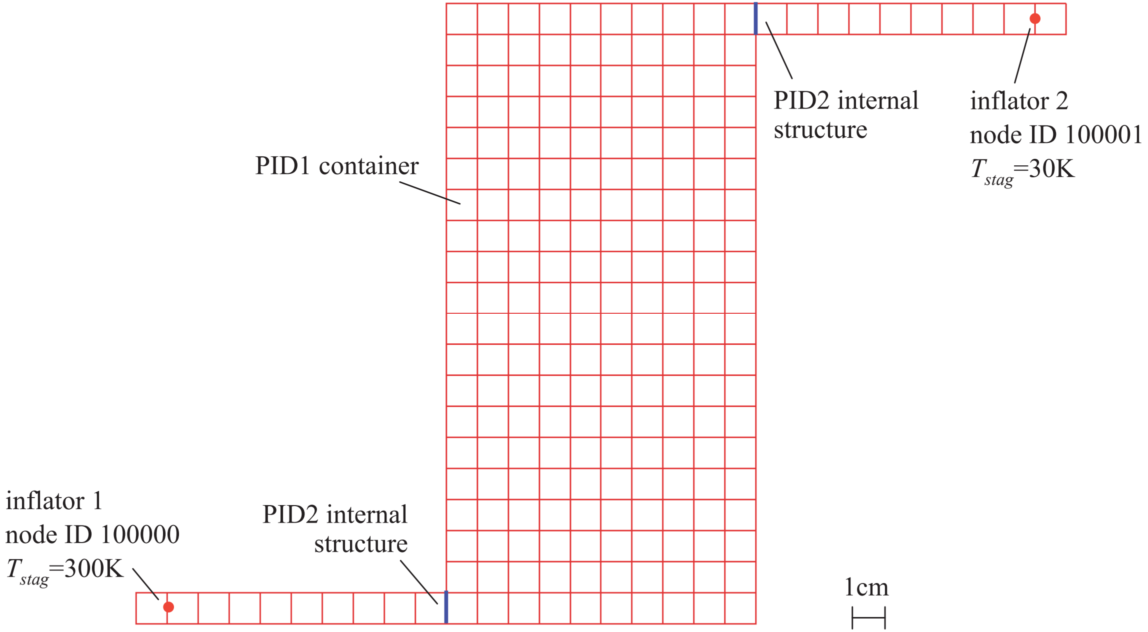
Gas pressure against moving wall versus time.



Numerical examples

Multiple inflators

Simple model with two inflators shooting in air at different temperatures. An internal structure (PID2) blocks the gas from time 0 to 2ms.



Numerical examples

Multiple inflators

Gas components can be assigned to be shot out through specific orifices. By default the gas is distributed to all orifices (if no inflator ID is given).

***AIRBAG_PARTICLE**

12, 1, 2, 0	Container + internal membrane is part set ID 12, membrane is PID2
20000, 1, 1	20k particles, SI-units and visible particles
0, 2, 2	No initial air, 2 gas components and 2 orifices
1, 2, 0.028, 28.0, 0, 0, 1	Air 1: mass flow curve ID=1, Temperature curve ID=2, inflator ID=1
3, 4, 0.028, 28.0, 0, 0, 2	Air 2: mass flow curve ID=3, Temperature curve ID=4, inflator ID=2
100000, 1.0, 0, 0, 1	Node ID 100000 represents orifice 1, inflator ID=1
100001, 1.0, 0, 0, 2	Node ID 100001 represents orifice 2, inflator ID=2

```
$Mass flow curve 1
*DEFINE_CURVE
1
0.0,        1.0
0.9e-4,     1.0
1.1e-4,     0.0
1.0,        0.0
```

```
$Static temp 1=300/ γ_Air
*DEFINE_CURVE
2
0.0,    210.9
1.0,    210.9
```

```
$Mass flow curve 2
*DEFINE_CURVE
3
0.0,        0.0
0.9e-4,     0.0
1.1e-4,     1.0
1.9e-4,     1.0
2.1e-4,     0.0
1.0,        0.0
```

```
$ Static temp 2=30/ γ_Air
 *DEFINE_CURVE
4
0.0,    21.09
1.0,    21.09
```

Numerical examples

Multiple inflators

FLC and FAC curves are defined for the membrane. It acts like a valve, opening up at time 2ms.

```
*MAT_FABRIC
MID       RO        EA        EB        EC        PRBA      PRCA      PRCB
GAB       GBC       GCA       CSE       EL        PRL       LRATIO    DAMP
AOPT      -5        -6        ELA       LNRC      FORM      FVOPT     TSRFAC
                              A1        A2        A3
V1        V2        V3        D1        D2        D3        BETA
LCA       LCB       LCUA      LCUB      LCUAB
```

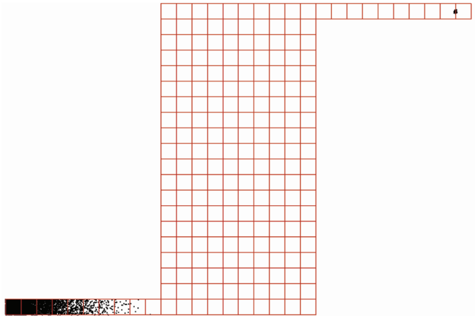
```
$Leakage area scale factor versus time FAC
*DEFINE_CURVE
5
0.0,      0.0
1.9e-3,   0.0
2.1e-3,   1000.0
1.0,      1000.0
```

```
$Leakage velocity FLC
*DEFINE_CURVE
6
0.0,   1000.0
1.0,   1000.0
```

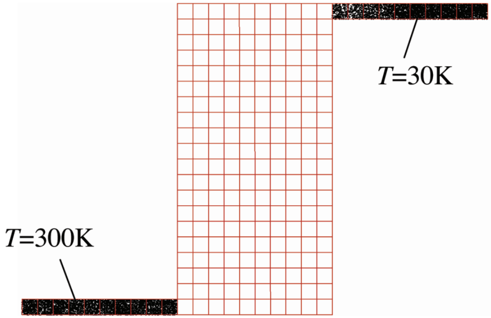
Numerical examples

Multiple inflators

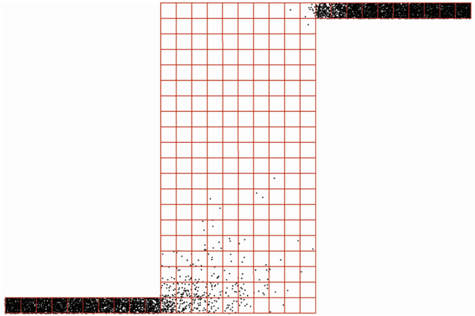
t=0.1ms



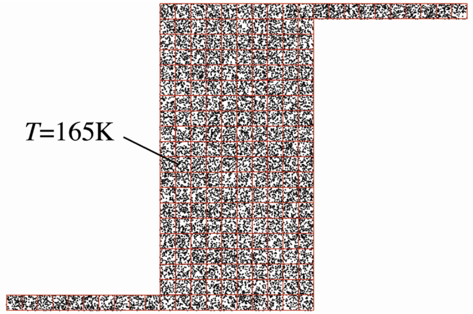
t=2.0ms



t=2.1ms



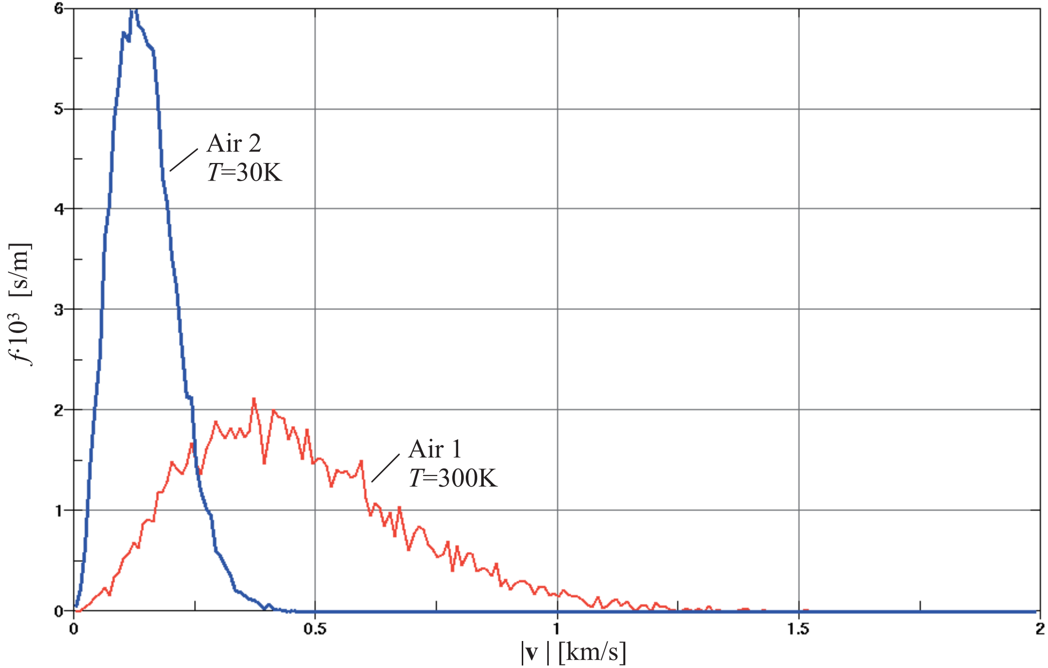
t=100ms



Numerical examples

Multiple inflators

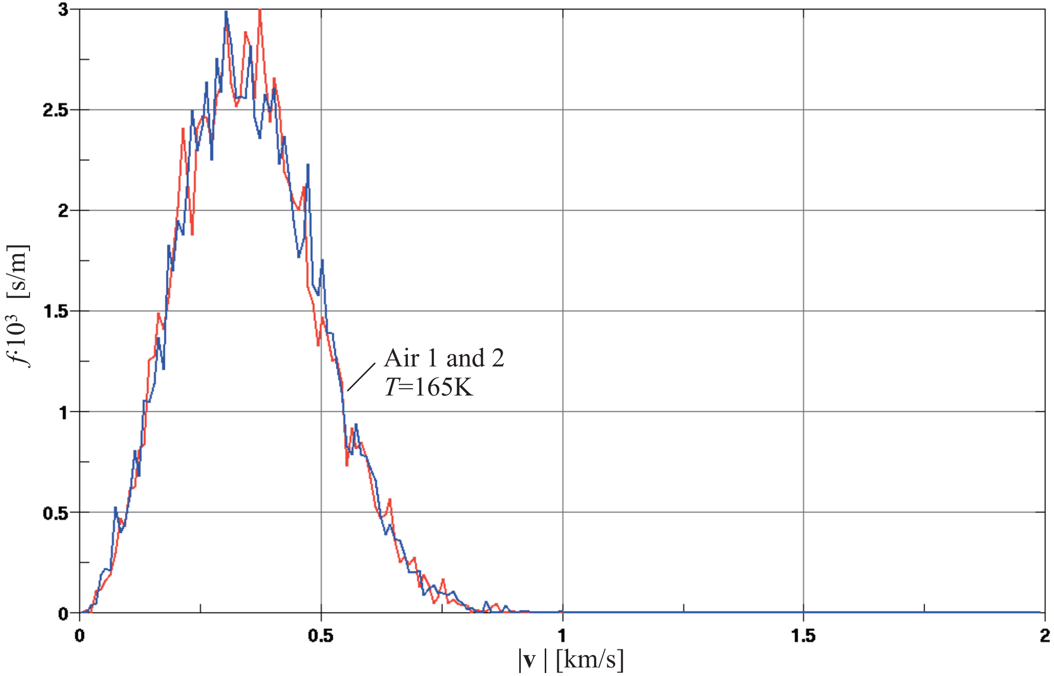
Velocity distribution of air particles before mixing, at time 2ms.



Numerical examples

Multiple inflators

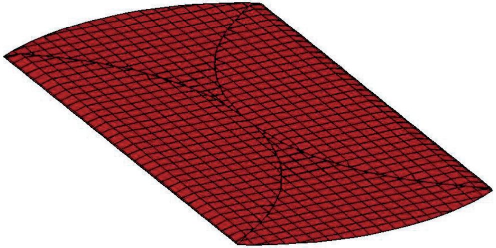
Velocity distribution of air particles after mixing, at time 100ms.



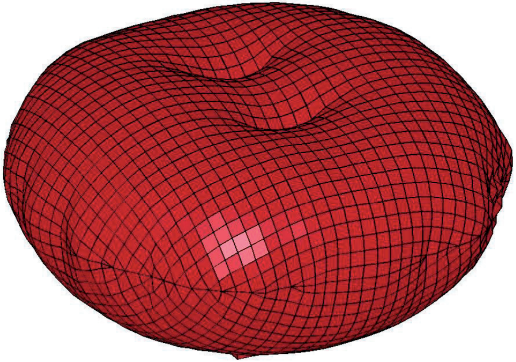
Numerical examples

Airbag

A simple driver bag is deployed with 100k particles. There is a switch to a control volume formulation after 15ms and the termination time is 30ms. The total CPU time on an AMD Athlon 4200 was 7min 10s.



folded bag at t=0



deployed bag at t=30ms

Numerical examples

Airbag

Gas components can be assigned to be shot out through specific orifices. By default the gas is distributed to all orifices (if no inflator ID is given).

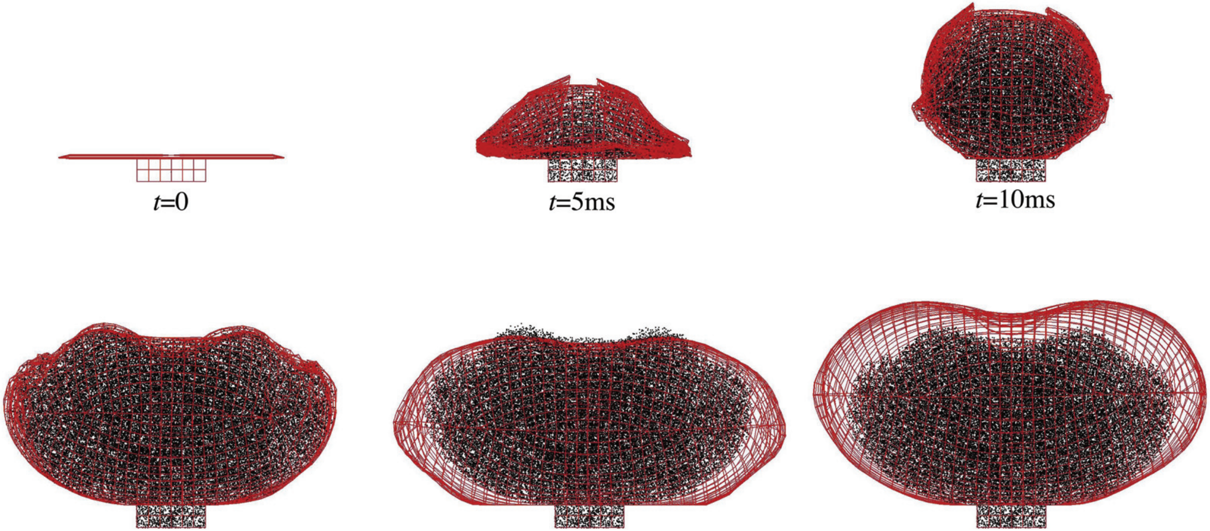
***AIRBAG_PARTICLE**

333, 1, 56, 1	All parts in set ID 333, internal parts (tethers) in set ID 56
100000, 0, 1, 0.0, 0.0, 0, 15.0, 15.0	100k particles, units kg-mm-ms-K, visible particles, switch to CV at 15ms
1, 1, 1	Initial air considered, 1 gas component and 1 orifice
0,0, 0,0, 0.02897, 28.0	Default pressure and temperature of initial air (1bar, 293K)
10, 20, 0.02897, 28.0	Inflator gas: mass flow curve ID=10, Temperature curve ID=20
1000000, 1.0	Node ID 1000000 represents the orifice

Numerical examples

Airbag

Snapshots at different times. The particles freeze in their current location when switching to the CV-formulation.



t=0

t=5ms

t=10ms

t=15ms
switch to CV-formulation

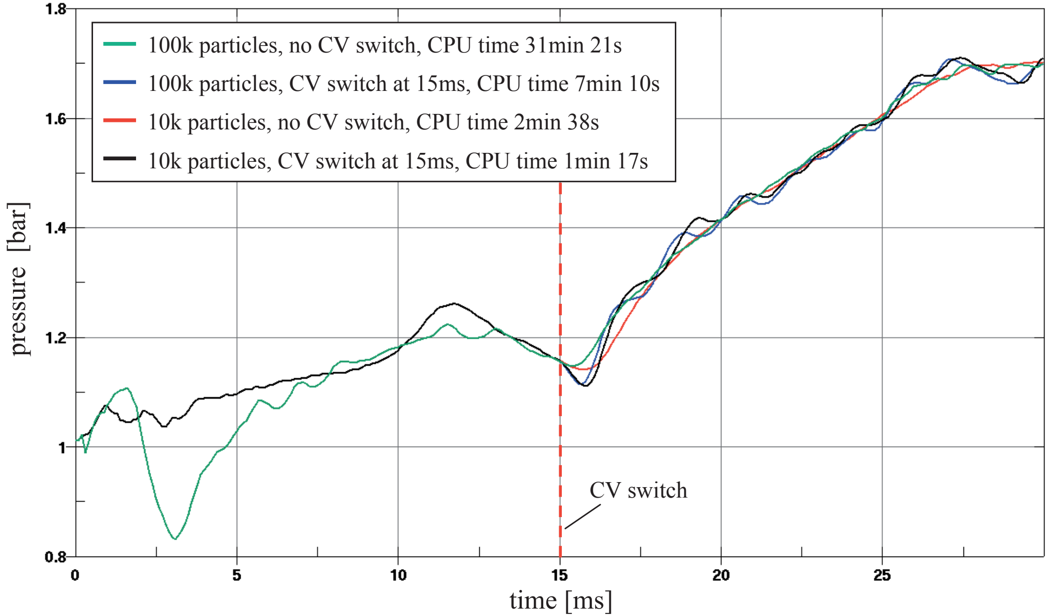
t=20ms

t=30ms

Numerical examples

Airbag

Bag pressure versus time using different number of particles and with/without a CV-switch.



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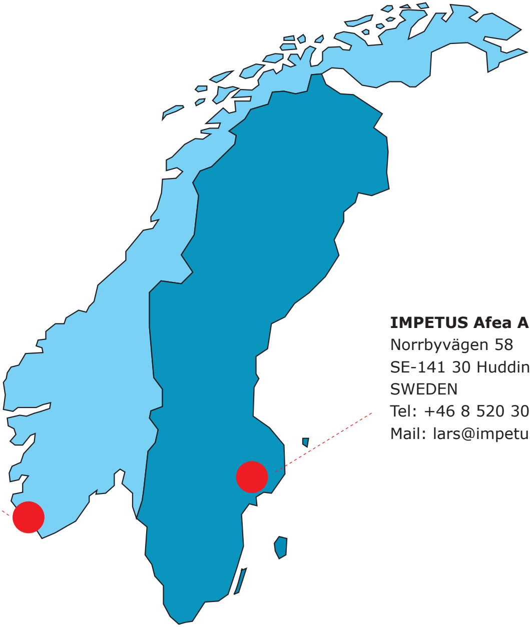

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