

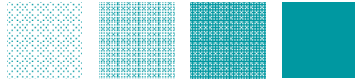
Explizite Finite Elemente Methode

LV02: Masterkurs für MK-M, ME-M und PE-M

Numerische Simulation
FEM versus PFC und CFD

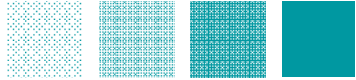


Prof. Dr.-Ing. Hans-Dieter Kleinschrodt
FB VIII: Maschinenbau, Veranstaltungstechnik, Verfahrenstechnik

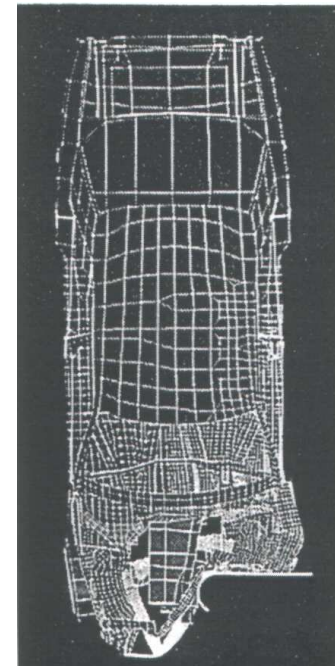
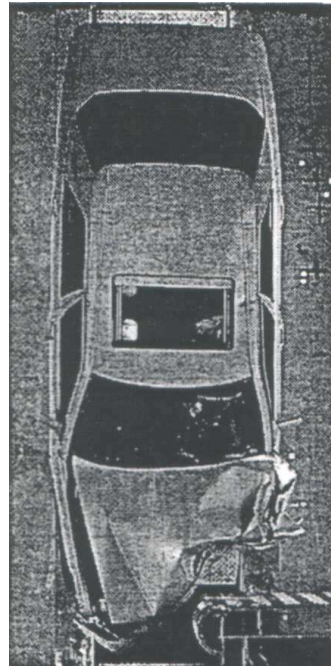


- **FEM** Finite Elemente Methode
- **PFC** Partical Flow Code
- **CFD** Computational Fluid Dynamics



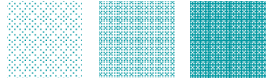


Wirklichkeit: technisches Problem (Versuch)



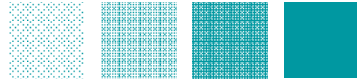
Simulation: wirklichkeitsgetreu nachahmen
virtuell mit numerischen Methoden





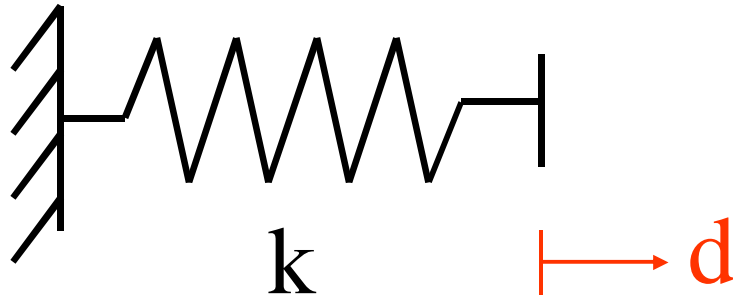
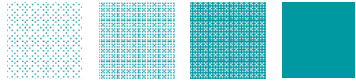
- Risschadensanalyse im Rohrboden eines Wärmetauschers (FEM)
- Ursachenanalyse für Entmischungen in einem Schacht (PFC)
- Ausgasung einer Flüssigkeit in einem elektrochemischen Reaktor (CFD)



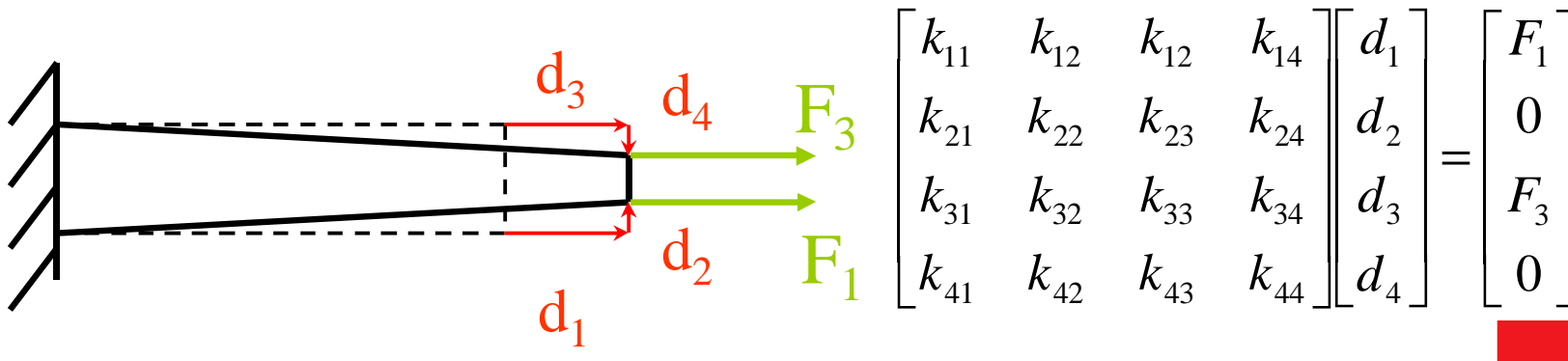
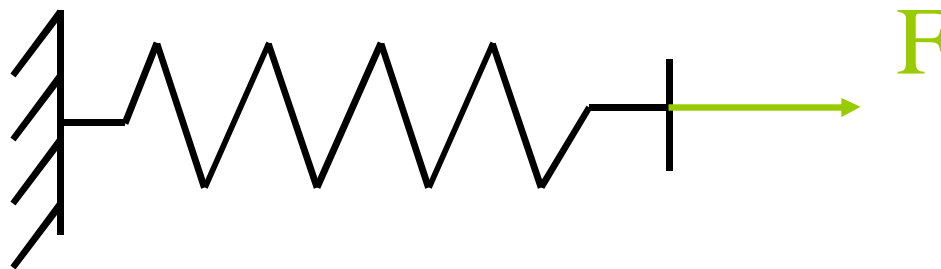


- **Verständnis des Vorgangs**
(Nachbildung der Natur)
- **Optimierung des Vorgangs**
- **Vorhersage des Vorgangs**

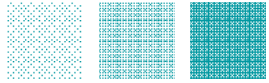




$$kd = F$$

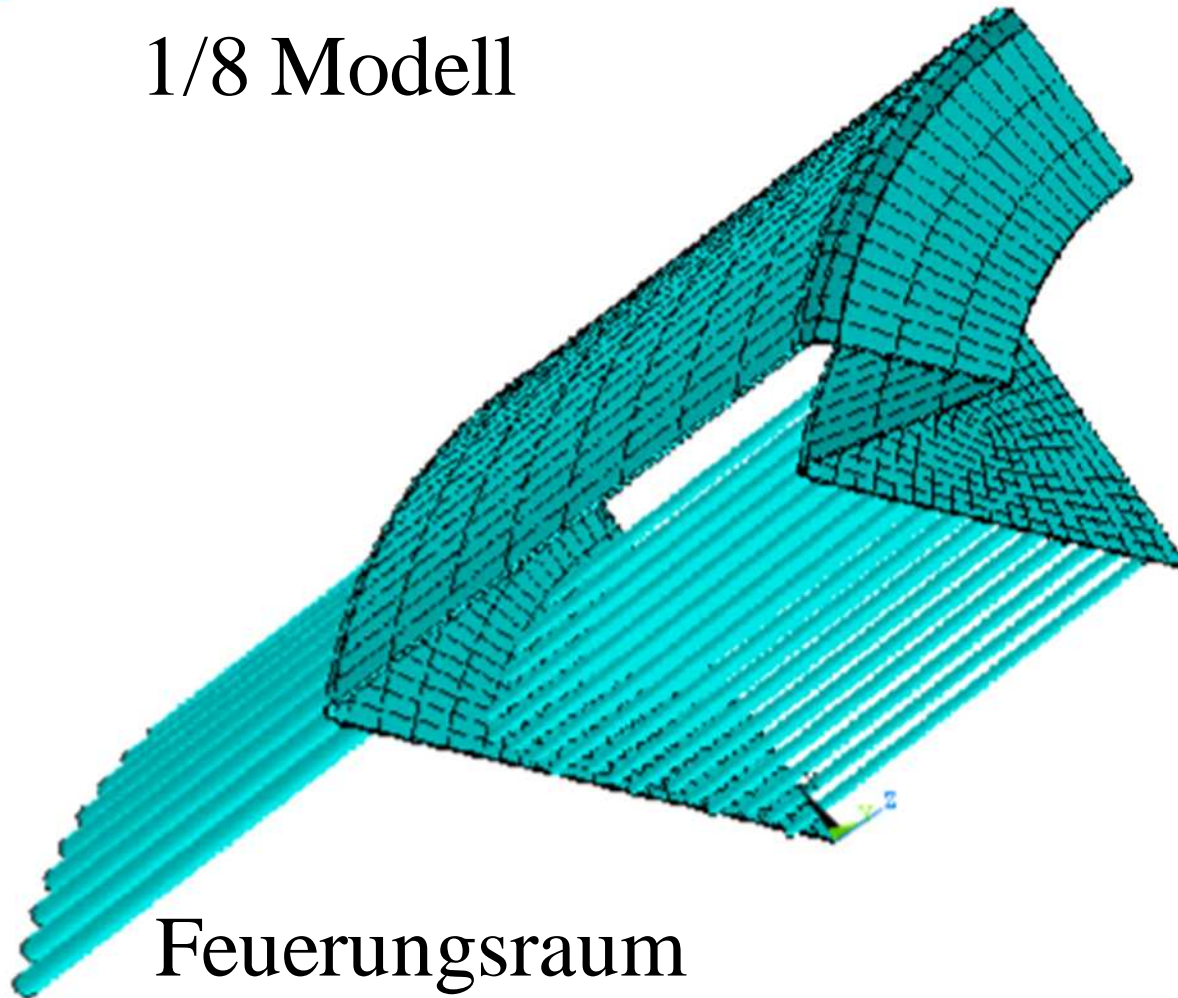


$$\begin{bmatrix} k_{11} & k_{12} & k_{12} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}
 \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}
 =
 \begin{bmatrix} F_1 \\ 0 \\ F_3 \\ 0 \end{bmatrix}$$



1

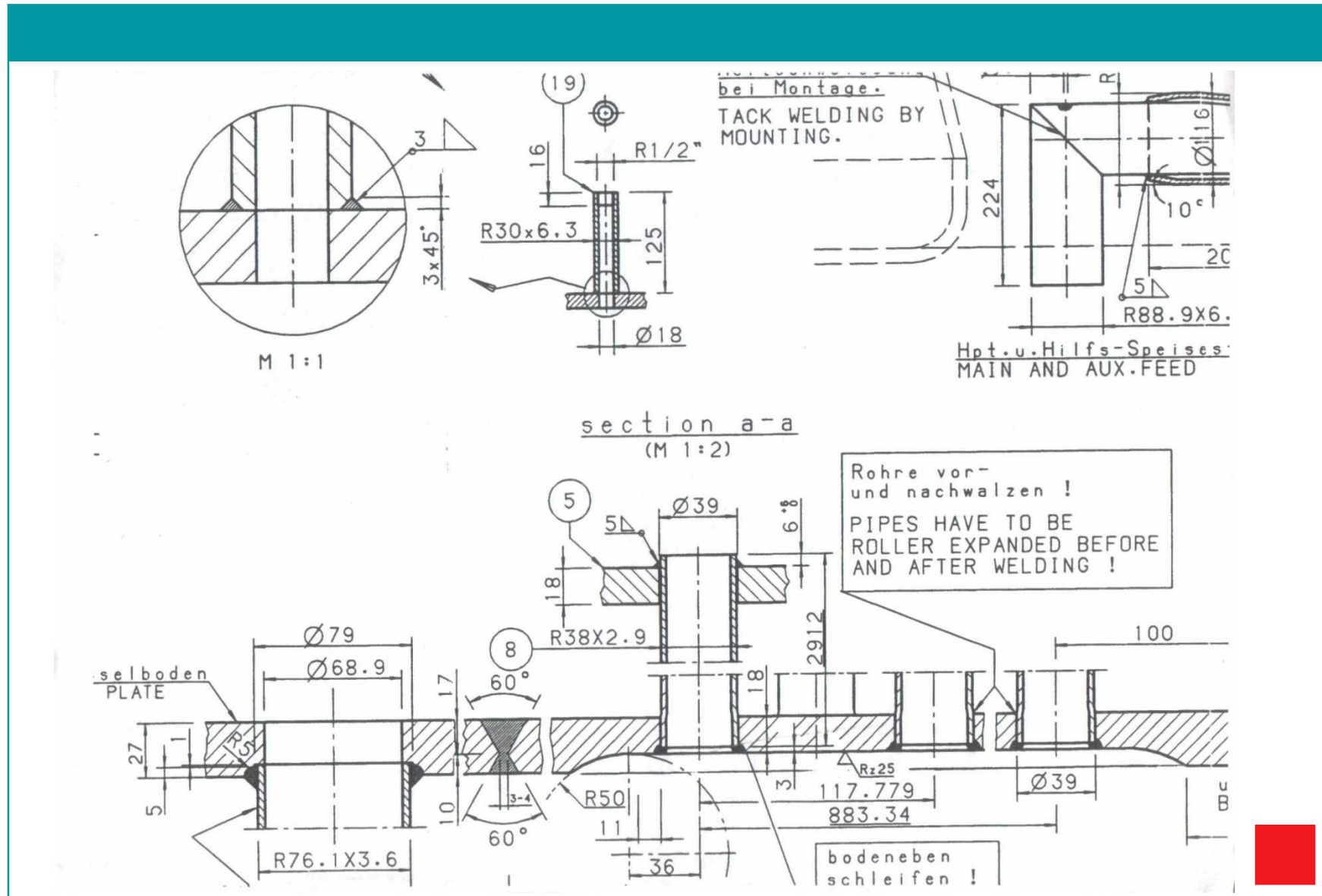
1/8 Modell

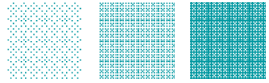


Feuerungsraum



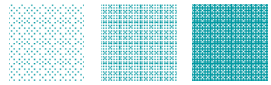
Boiler mit Rauchrohren als Wärmetauscher



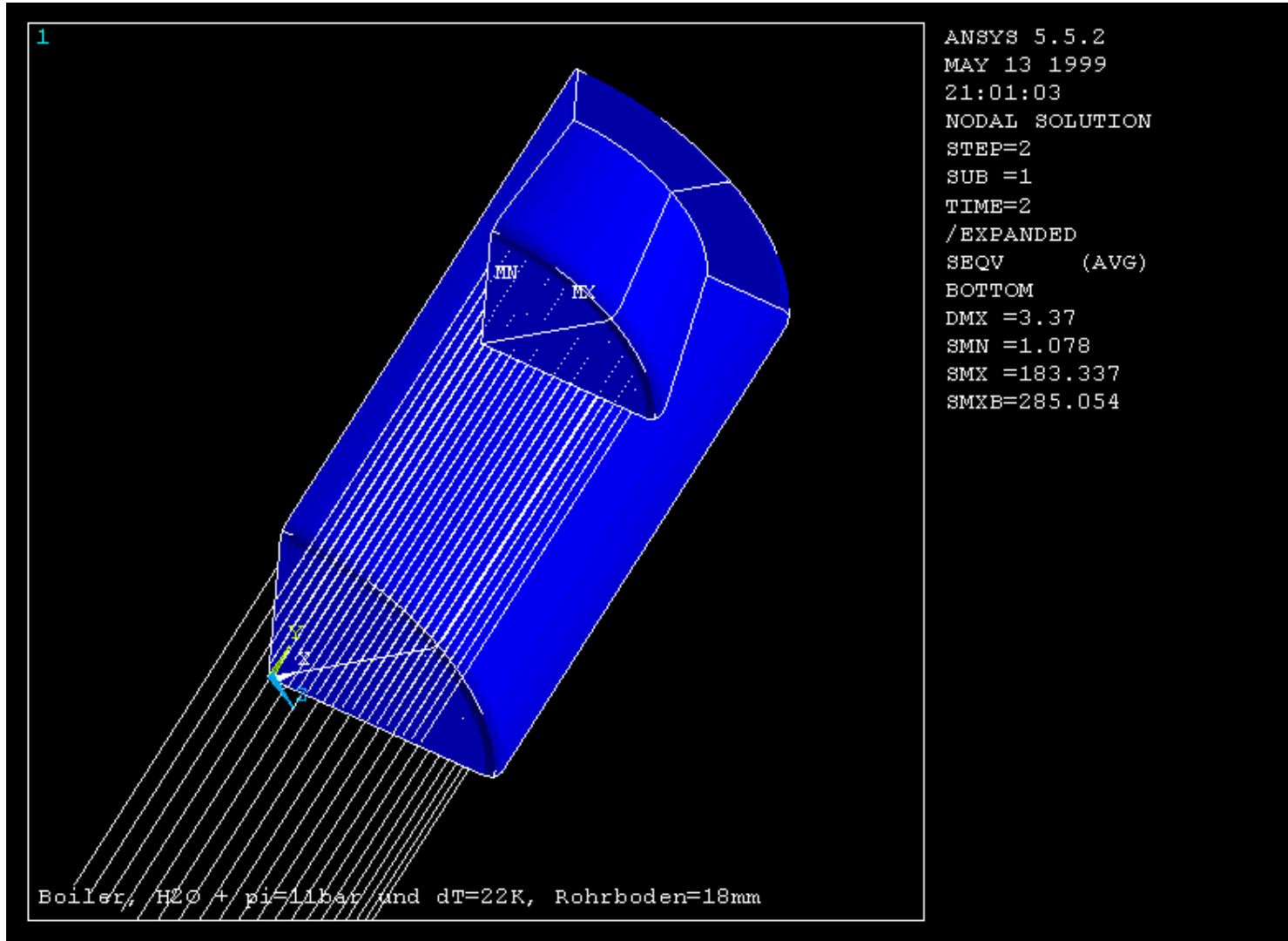


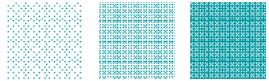
- linear elastisches Materialverhalten
- kleine Verschiebungen
- Schalen- und Balkenelemente
- 2000 Knoten mit je 6 Freiheitsgraden (DOF)
 - 3 Verschiebungen und 3 Drehungen
- Lineares Gleichungssystem mit
 - 12 000 DOF
- Belastung:
 - Innendruck + Temperaturdifferenz



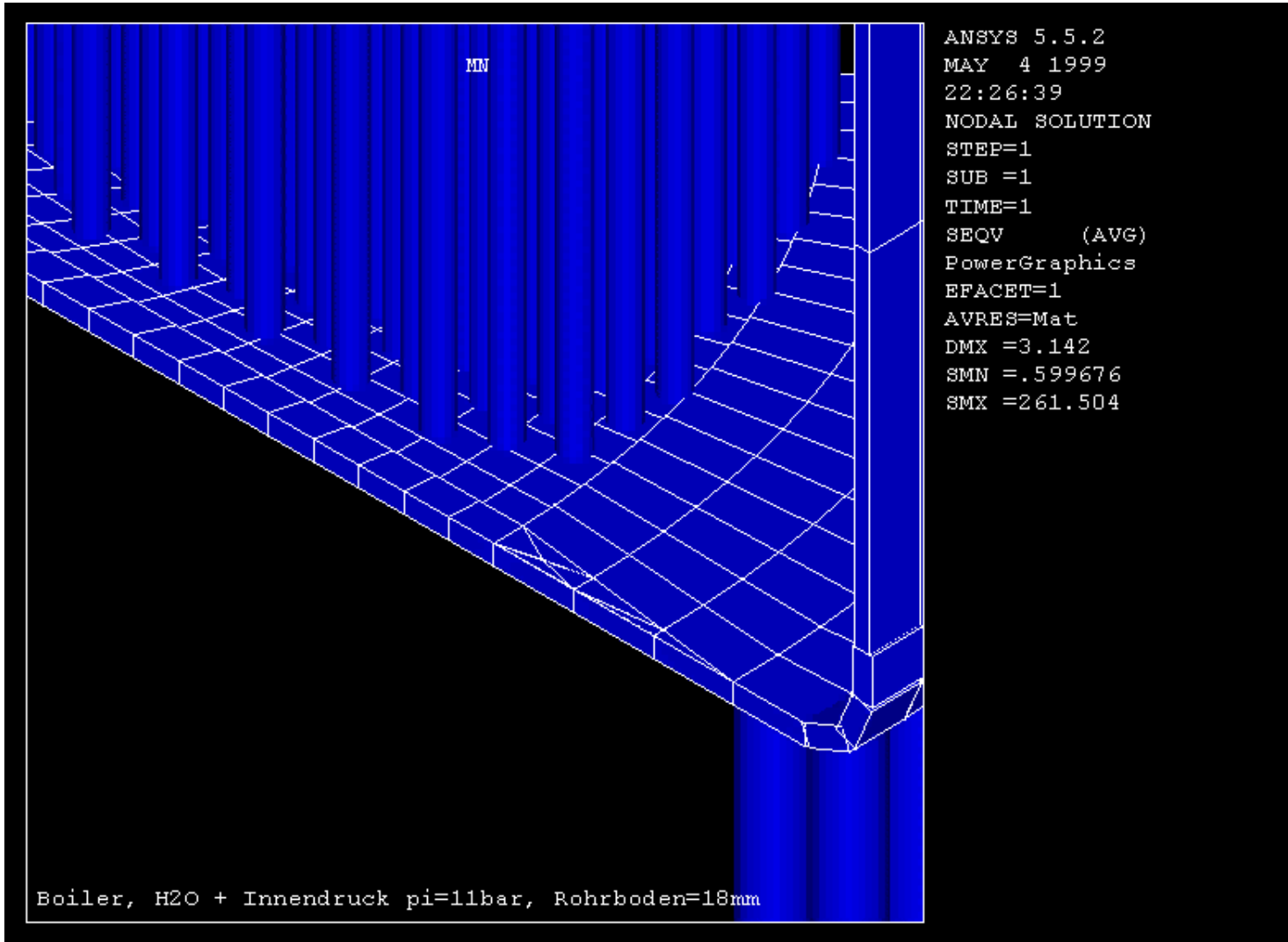


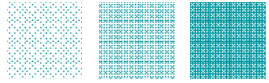
Belastung: Druck und Temperatur



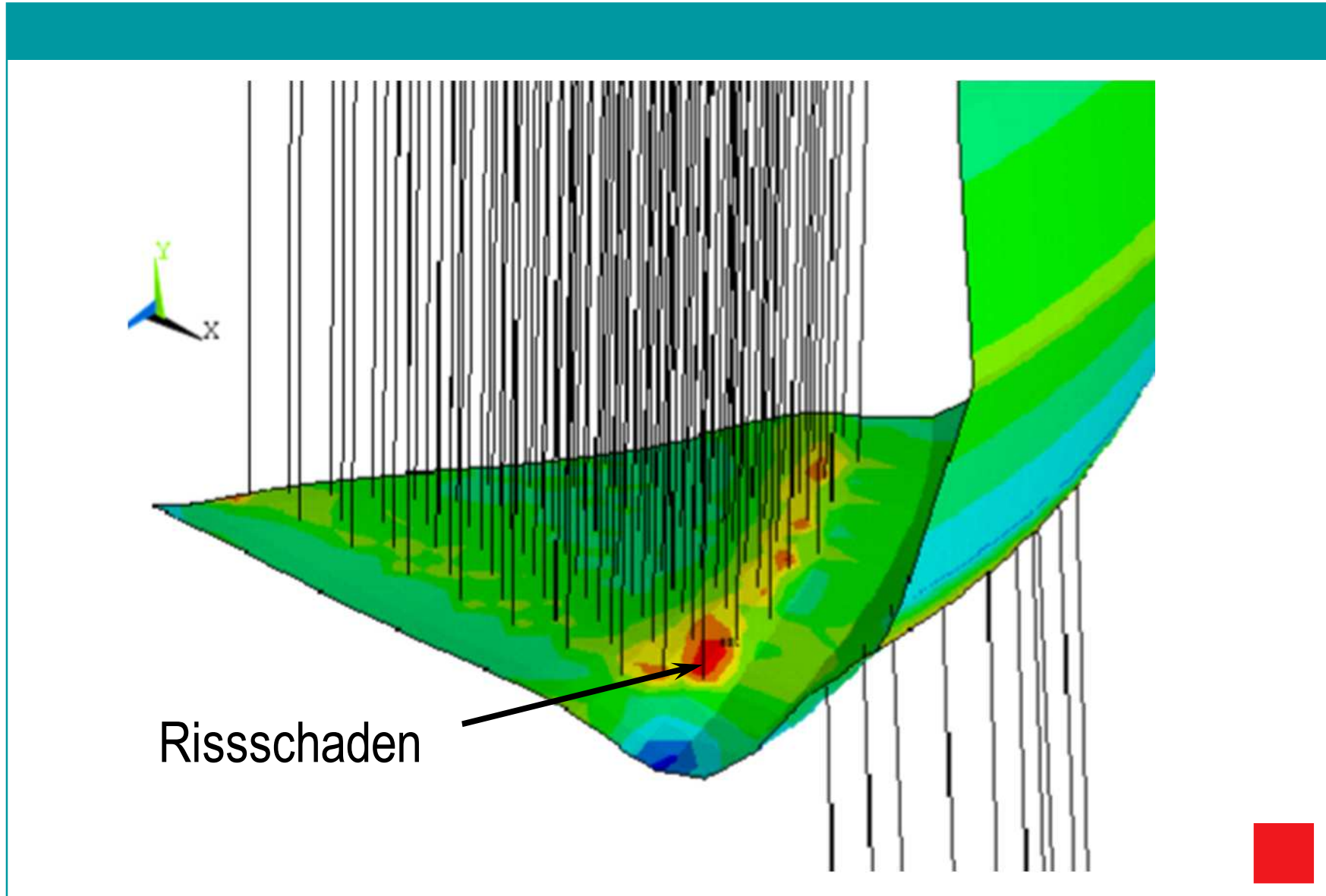


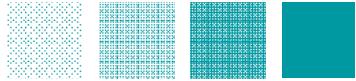
Vergleichsspannungen nach von Mises





Vergleichsspannungen im Rohrboden





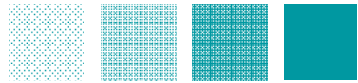
vor

nach

m Masse
 J_S Massenträgheitsmoment

S

Kontakt glatt oder rauh
(Gewichtskraft vernachlässigt)



■ Experiment

- Ball fällt zwischen 2 schiefen Ebenen runter ! ... oder nicht ?

■ Simulation

- FEM mit glattem Kontakt

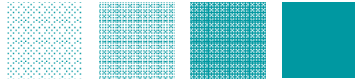


- FEM mit rauem Kontakt

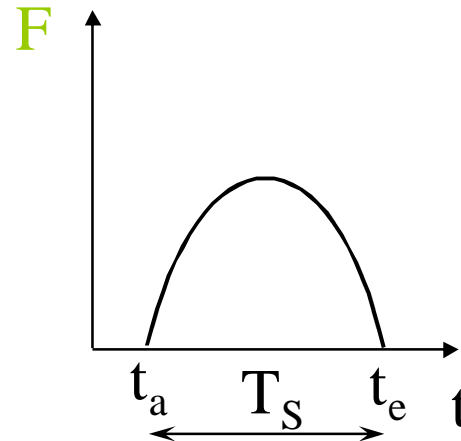
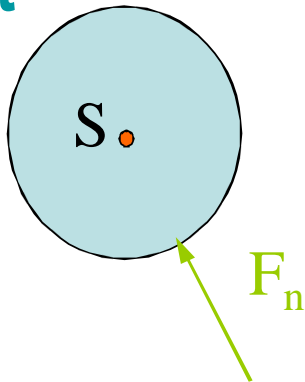


- PFC mit 1 oder 2 Bällen

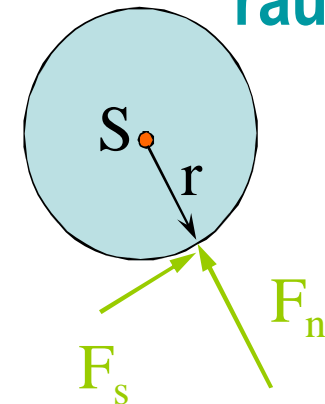




glatt



rauh



Impulssatz:
(I. Newton)

$$\sum \vec{F}_i = m \dot{\vec{v}}$$

Drallsatz (eben):
(L. Euler)

$$\sum M_i^S = J_S \dot{\omega}$$

Zeitinte-
gration:

$$\int_{t_a}^{t_e} [\vec{F}_n(t) + \vec{F}_s(t)] dt = m \vec{v}_e - m \vec{v}_a$$

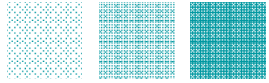
$$r \int_{t_a}^{t_e} F_s(t) dt = J_S \omega_e - J_S \omega_a$$

Energiesatz:

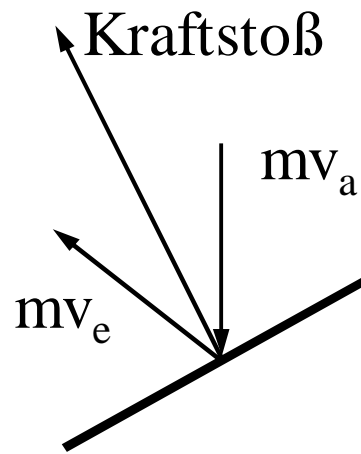
$$\frac{1}{2} m v_e^2 + \frac{1}{2} J_S \omega_e^2 = \frac{1}{2} m v_a^2 + \frac{1}{2} J_S \omega_a^2$$

(Stoß ohne
Verluste)

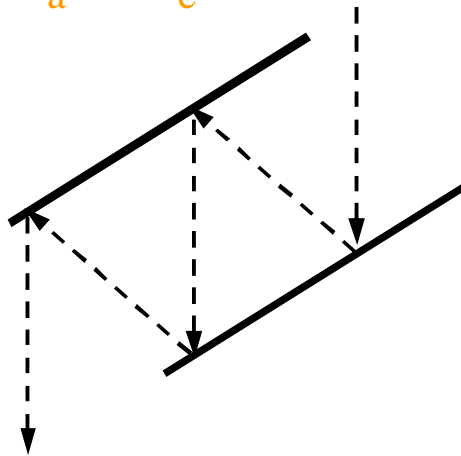




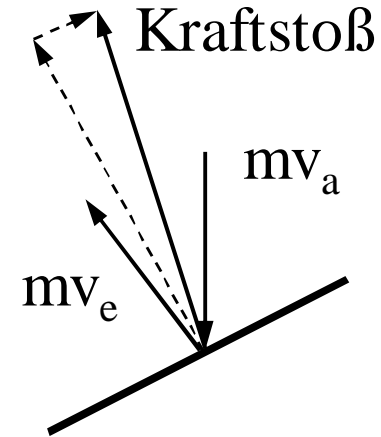
glatt:



$$\omega_a = \omega_e = 0$$

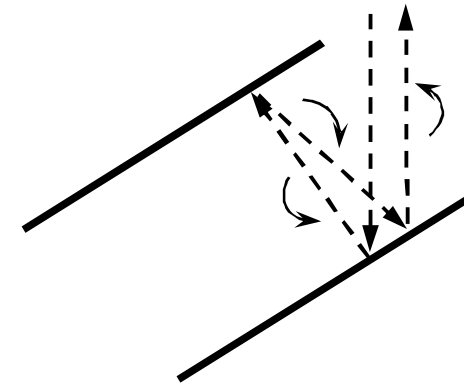


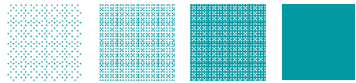
rauh:



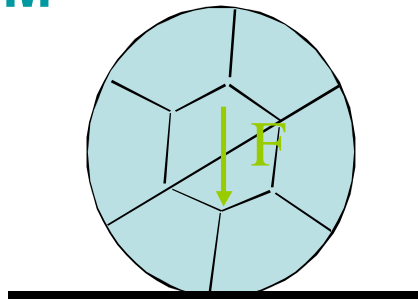
$$v_e < v_a$$

$$\omega_a = 0, \omega_e > 0$$



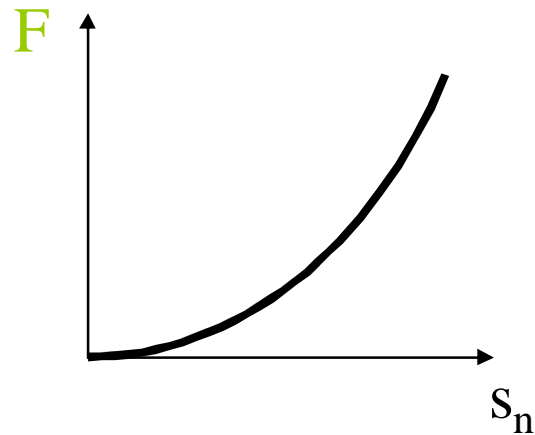
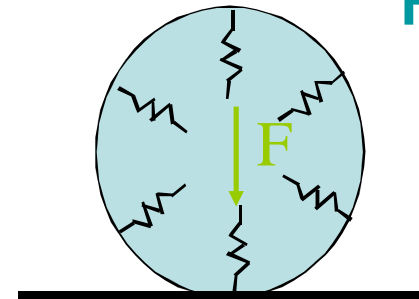


FEM

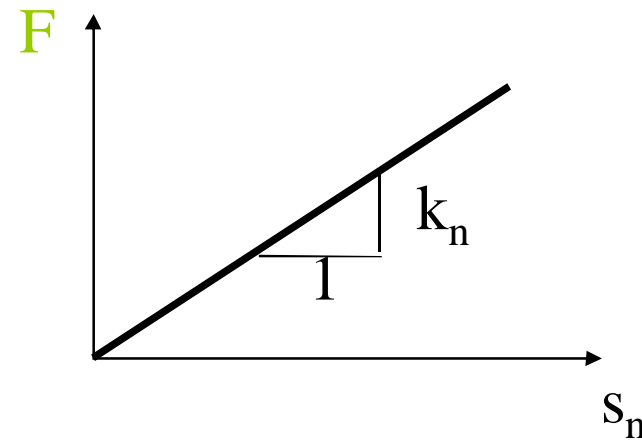


$\downarrow S_n$

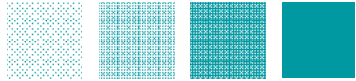
PFC



$$F \sim S_n^{3/2} \text{ (Hertz)}$$



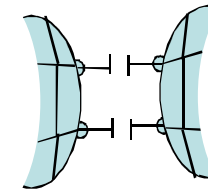
k_n Federsteifigkeit



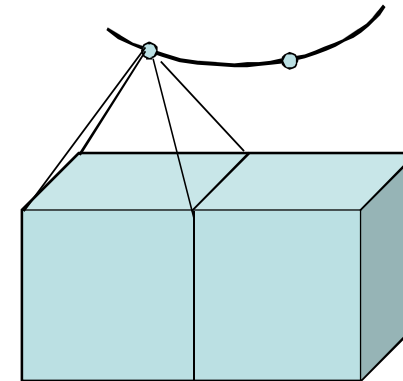
Kontakterkennung bei FEM



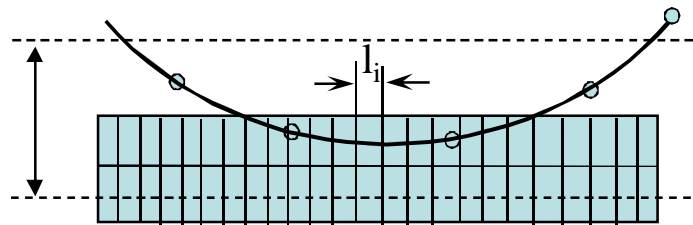
Knoten - Knoten



Knoten - Element



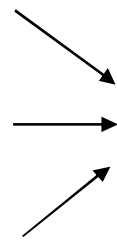
Überprüfungs-
bereich
 $10 \cdot l_i$



Linie - Linie

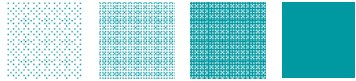
Linie - Fläche

Fläche - Fläche

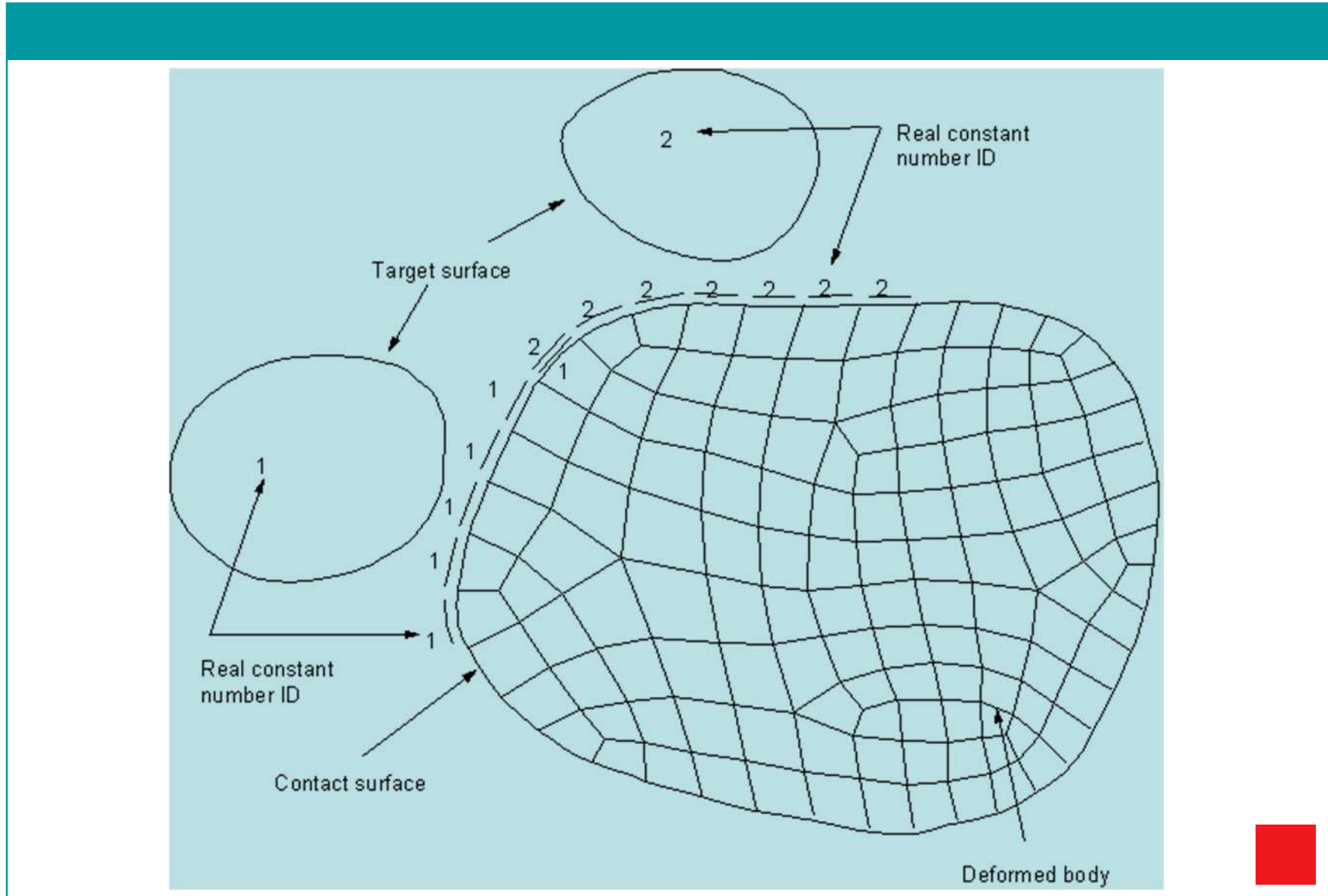


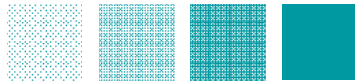
Kontakt-Wizard (Vorsicht Rechenzeit)



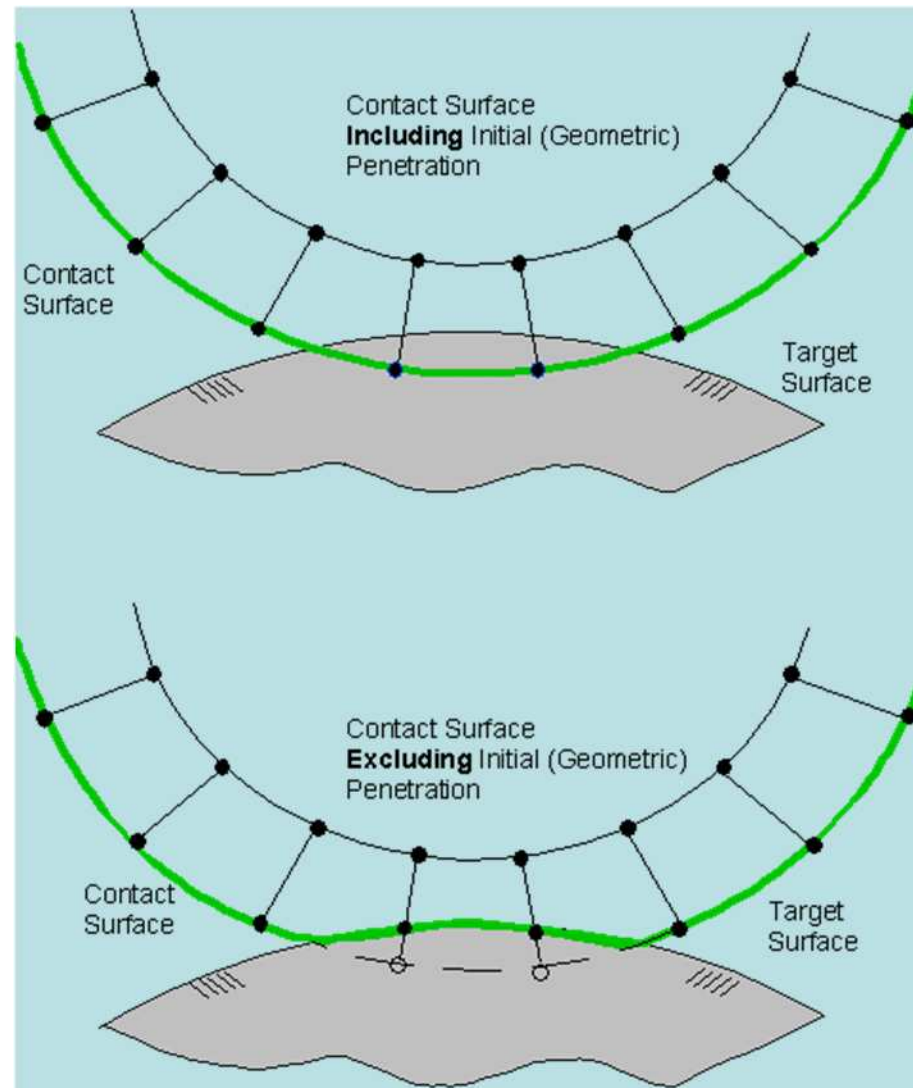


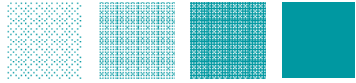
Kontakterkennung bei FEM





Kontakterkennung bei FEM



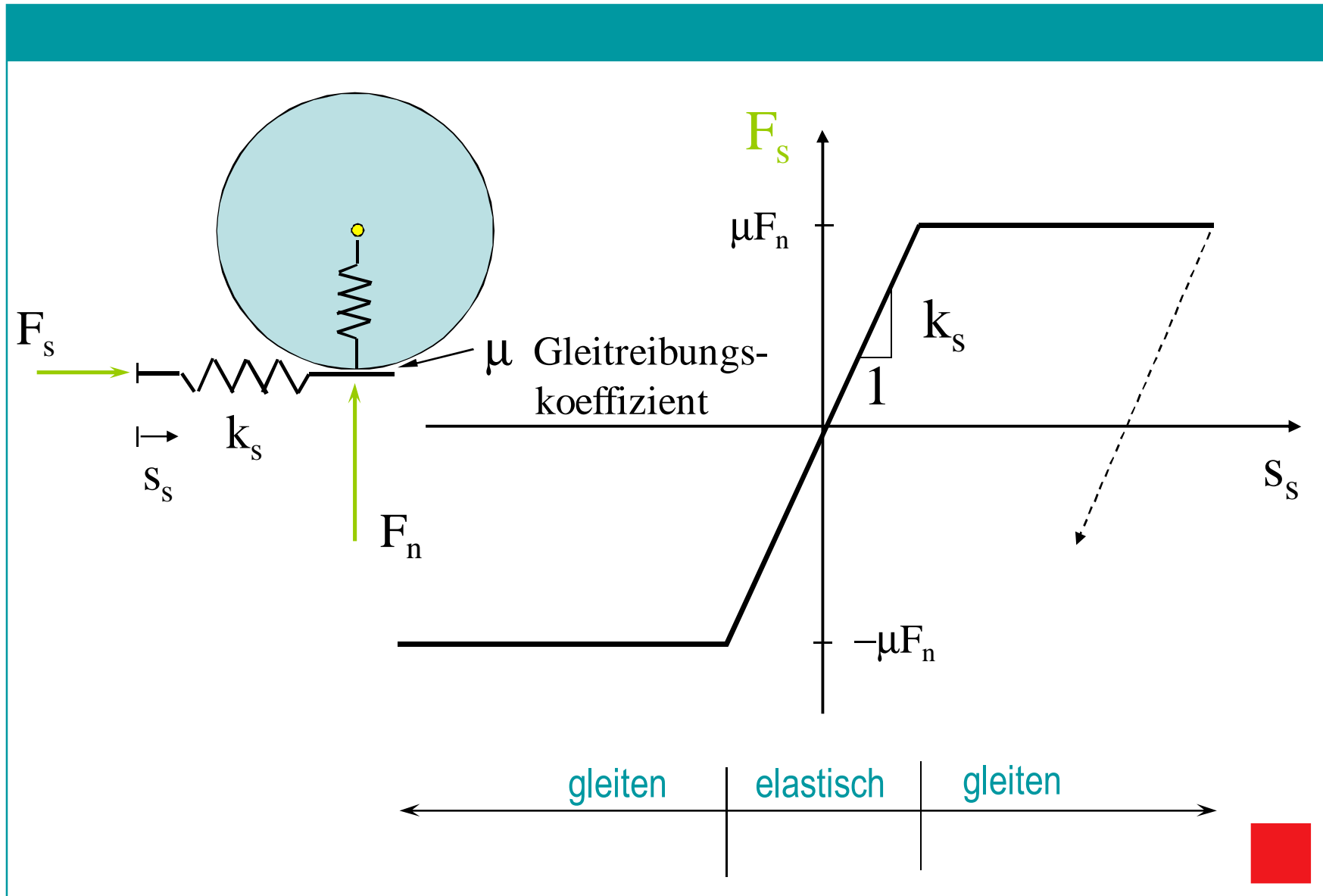
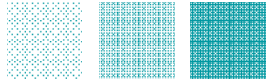


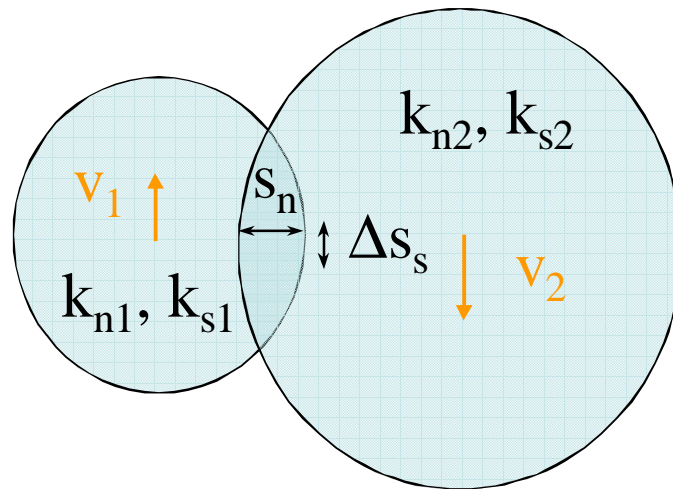
$a < r$

$a < r_i + r$

**explizites Zeit-
Integrations-
verfahren**

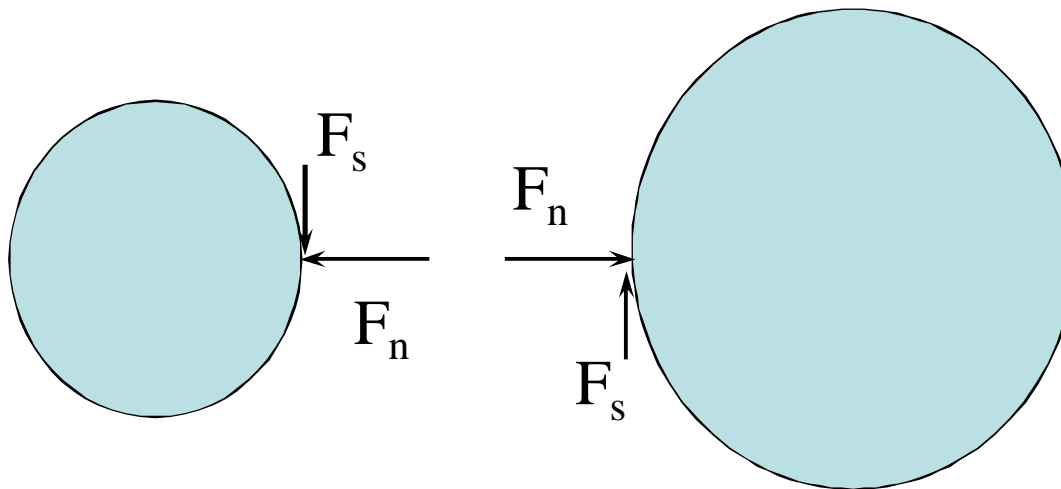
Überprüfung im Umfeld von $5 \cdot r_i$
des letzten Zeitschrittes





$$\frac{1}{k_n} = \frac{1}{k_{n1}} + \frac{1}{k_{n2}}$$

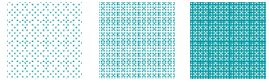
$$\frac{1}{k_s} = \frac{1}{k_{s1}} + \frac{1}{k_{s2}}$$



$$F_n = k_n s_n$$

$$\Delta F_s = k_s \Delta s_s$$

$$|F_s| \leq \mu |F_n|$$



Berechnungszyklus bei PFC



Aktualisierung der Partikel- und Wandpositionen sowie Kontakterkennung

Lösung der Bewegungsgleichungen für jeden Partikel infolge ΣF_i u. ΣM_i

Lösung der Kraft-Verschiebungsgesetze für jeden Partikel

Kontaktkräfte



für **ein** Partikel gilt:

$$\begin{bmatrix} m & & \\ & m & \\ & & J_S \end{bmatrix} \begin{bmatrix} \ddot{d}_x \\ \ddot{d}_y \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} \Sigma F_{ix} \\ \Sigma F_{iy} \\ \Sigma M_i^S \end{bmatrix}$$

dyn. Gleichgewicht zur Zeit t :

$$\underline{\underline{M}} \underline{\underline{\ddot{d}}}_t = \underline{\underline{f}}_t$$

Zentrale Differenzenformel:

$$\underline{\underline{\ddot{d}}}_t = \frac{1}{\Delta t^2} (\underline{\underline{d}}_{t-\Delta t} - 2\underline{\underline{d}}_t + \underline{\underline{d}}_{t+\Delta t})$$

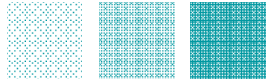
neue Lage:

$$\underline{\underline{d}}_{t+\Delta t} = \Delta t^2 \underline{\underline{M}}^{-1} \underline{\underline{f}}_t + 2\underline{\underline{d}}_t - \underline{\underline{d}}_{t-\Delta t}$$

explizites Zeitintegrationsverfahren

Stabilitätsbedingung:

$$\Delta t \leq \Delta t_{krit} = \min\left(2\sqrt{\frac{m}{k_n}}, 2\sqrt{\frac{J_S}{k_s}}\right)$$

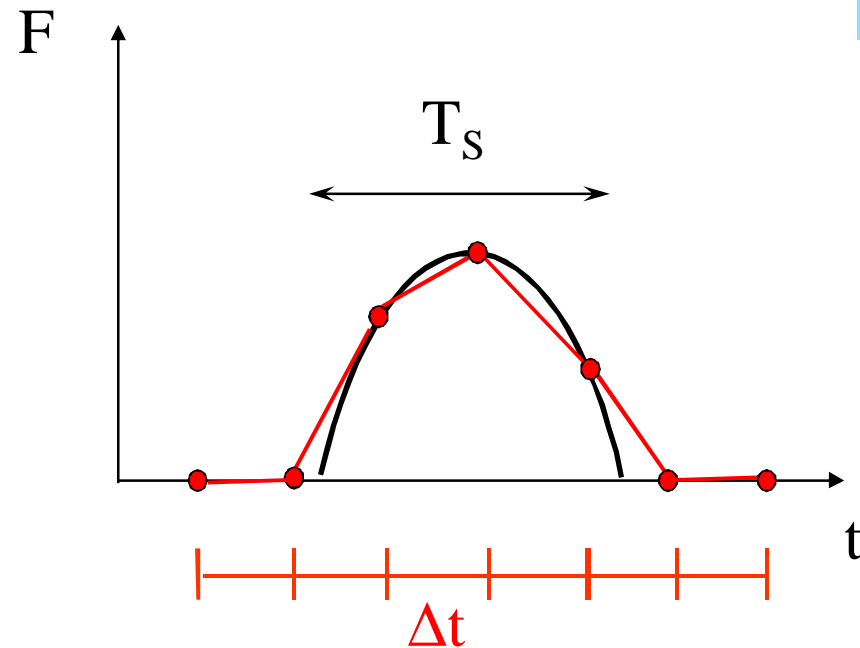


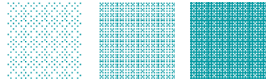
Eigenfrequenz: $f = \frac{1}{2\pi} \sqrt{\frac{k_n}{m}}$

Stoßzeit: $T_S = \pi \sqrt{\frac{m}{k_n}}$

Zeitschritt in PFC für die Integration:

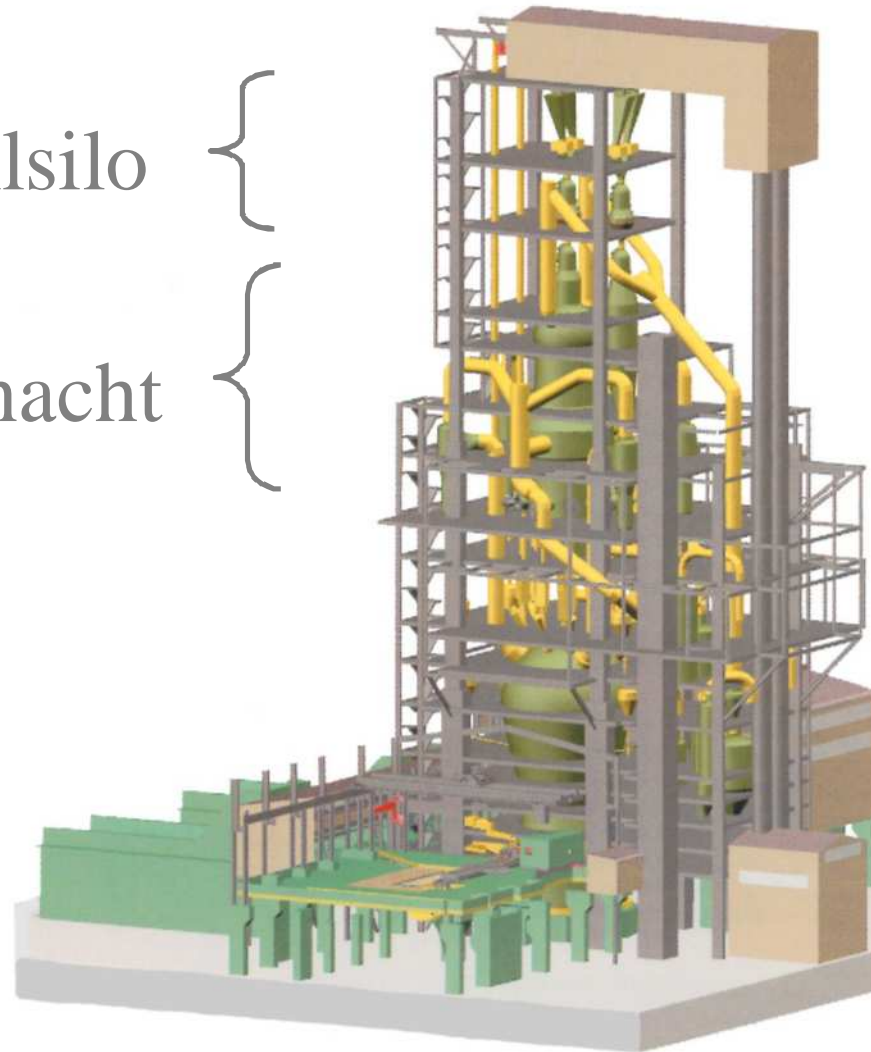
$$\Delta t = \sqrt{\frac{m}{k_n}}$$



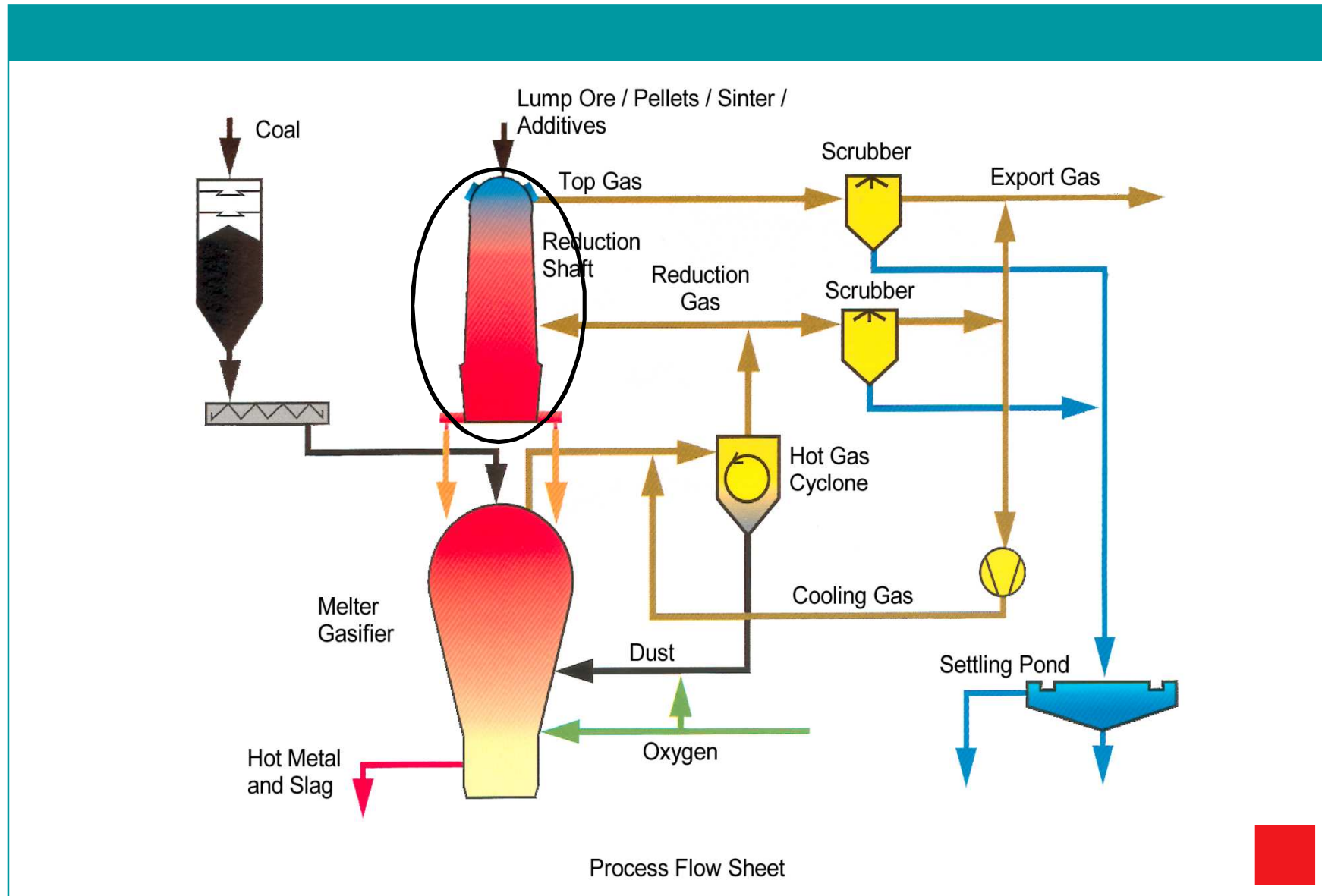


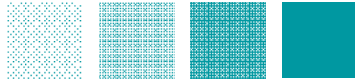
Füllsilo

Schacht



Fließschema eines COREX-Prozesses

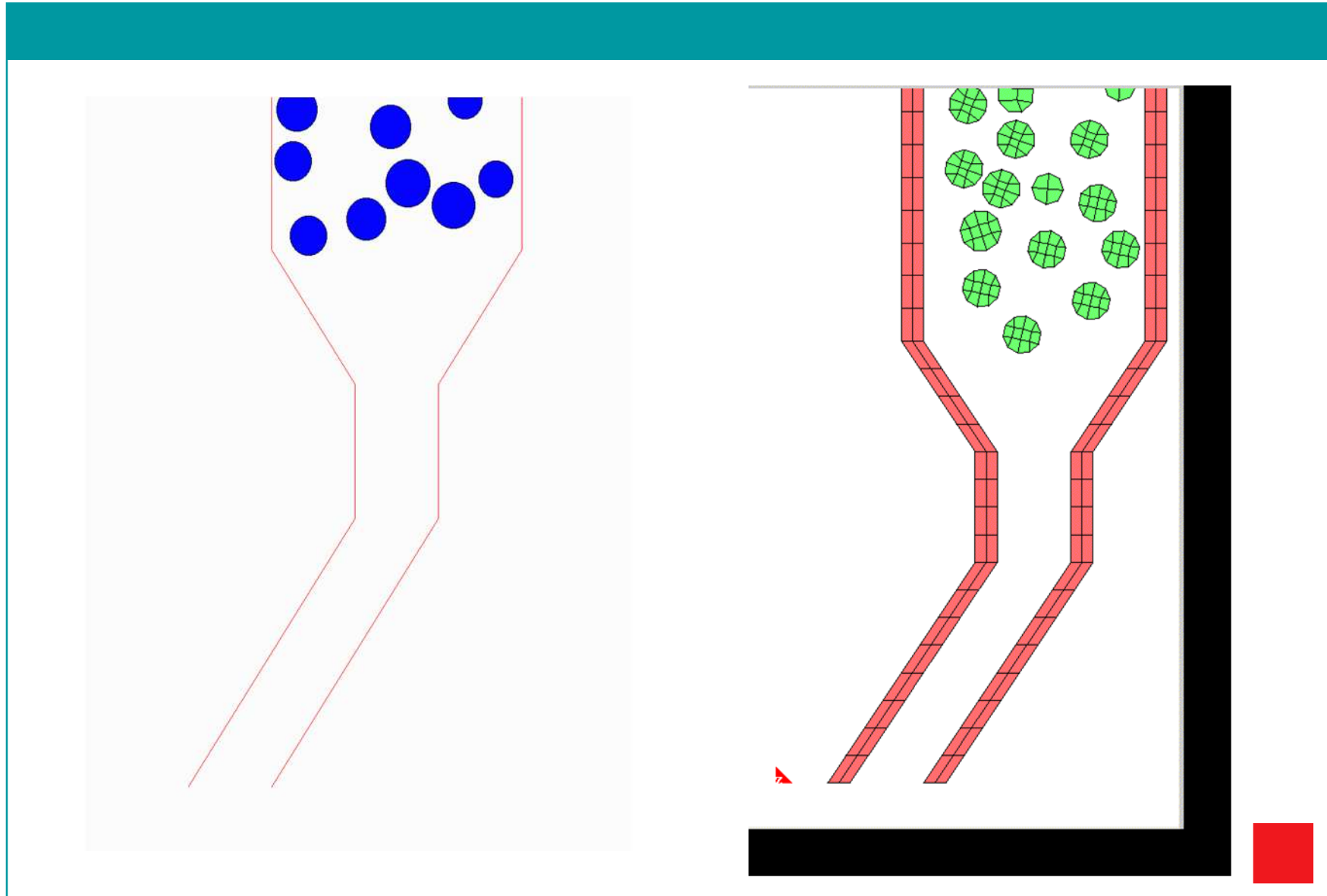


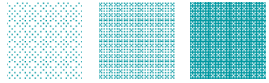


PFC

versus

FEM





$$\underline{\underline{M}} \underline{\underline{\ddot{d}}}_t + \underline{\underline{K}}(d) \underline{\underline{d}}_t = \underline{\underline{f}}_t$$

$$\underline{\underline{d}}_{t+\Delta t} = \Delta t^2 \underline{\underline{M}}^{-1} (\underline{\underline{f}}_t - \underline{\underline{K}}(d) \underline{\underline{d}}_t) + 2\underline{\underline{d}}_t - \underline{\underline{d}}_{t-\Delta t}$$

Massenmatrix $\underline{\underline{M}}$ i. a. nicht mehr Diagonalmatrix
Steifigkeitsmatrix $\underline{\underline{K}}$ nichtlinear, Aufbau in jedem Δt

Explizite FEM-Programmsysteme

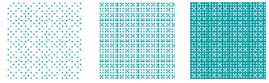
- LS-DYNA, ABAQUS Explizit u.a.

Implizite FEM-Programmsysteme

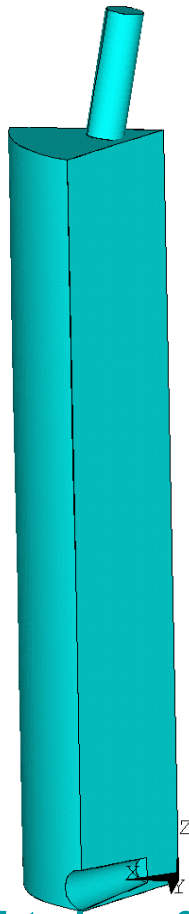
- ANSYS, NASTRAN, ABAQUS u.a.

dyn. Gleichgewicht
zur Zeit $t+\Delta t$:

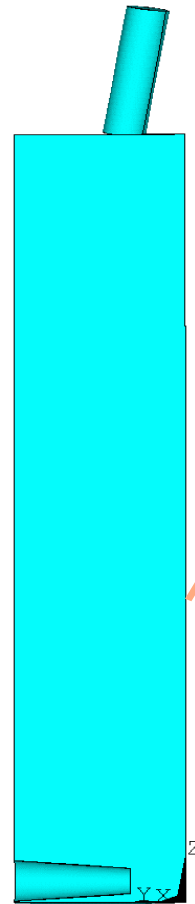
$$\underline{\underline{M}} \underline{\underline{\ddot{d}}}_{t+\Delta t} + \underline{\underline{K}}(d) \underline{\underline{d}}_{t+\Delta t} = \underline{\underline{f}}_{t+\Delta t}$$



2D-Idealisierungen eines 60° Ausschnittes

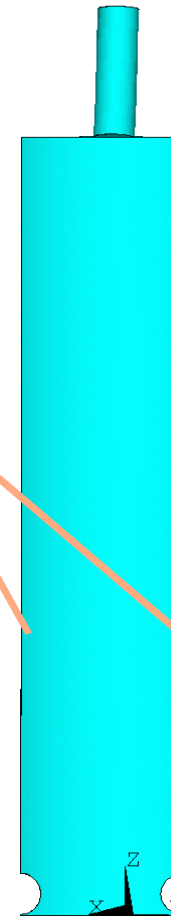


Ansicht eines 60°- Ausschnittes des Schachtes



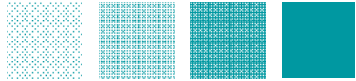
Radialschnitt

Symmetrische
Randbedingungen



Tangentialschnitt





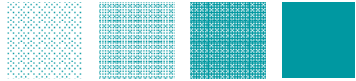
Stückerz



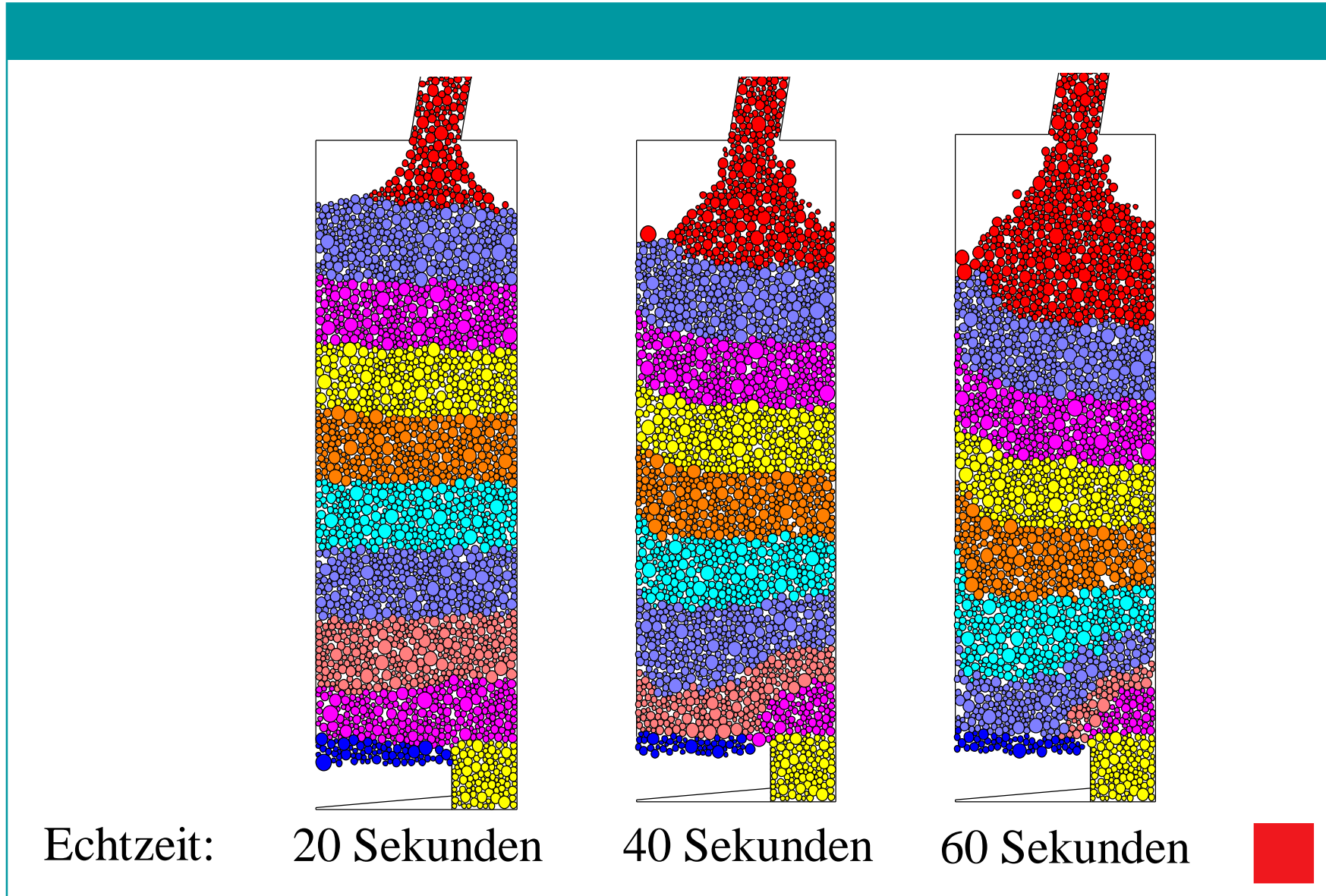
Dichte 3800 kg/m³ Innerer Reibungswinkel 45°

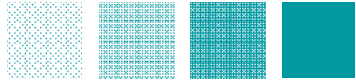
Massenstrom 150 t/h Größtkorndurchmesser 35 mm



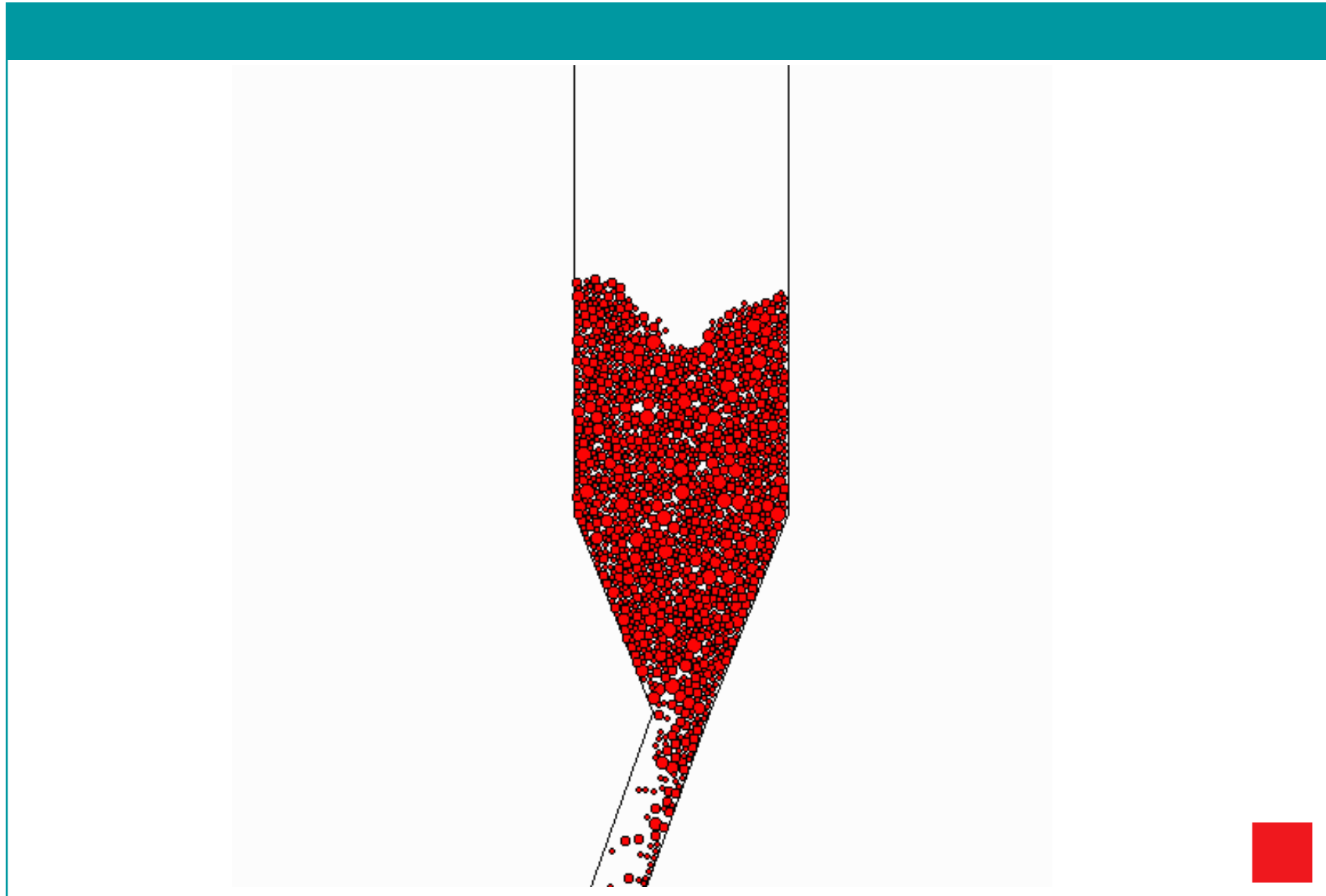


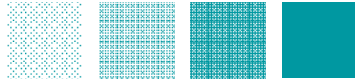
Zeitlicher Verlauf



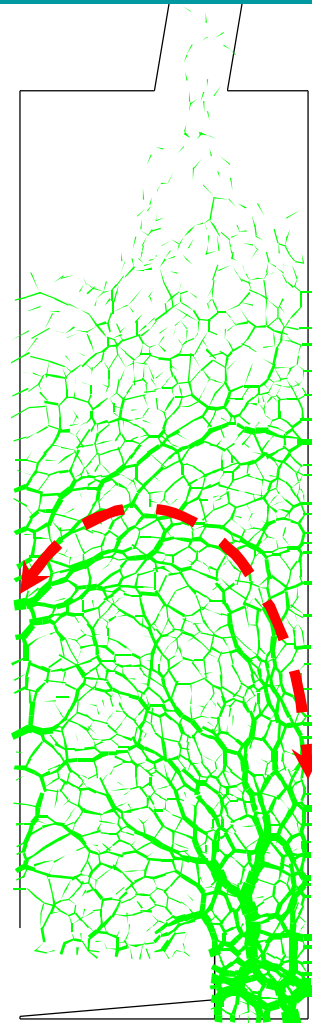


Animation Füllung und Entleerung

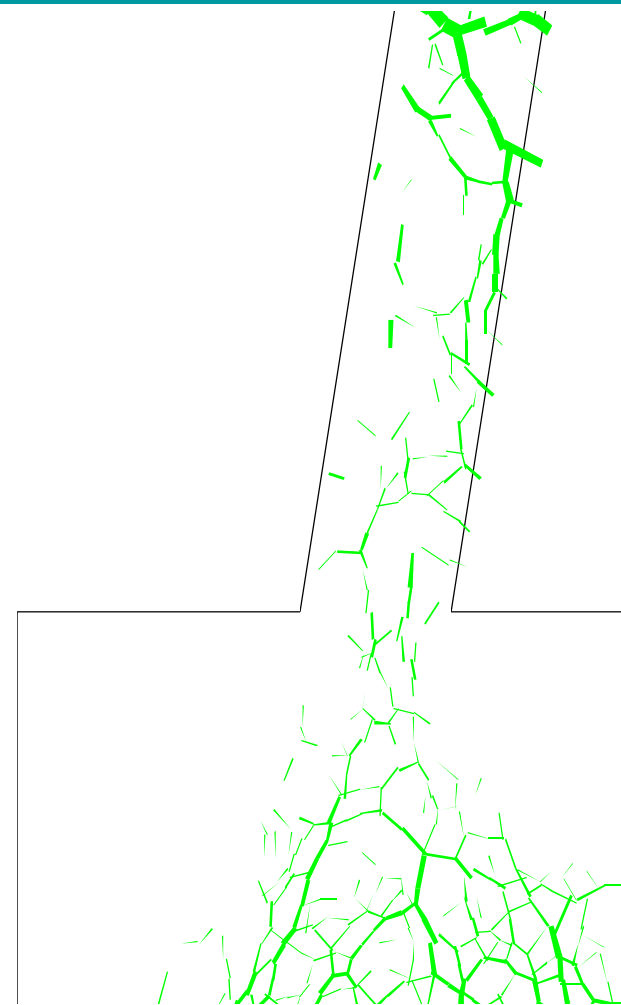




eventuell
Brücken-
bildung

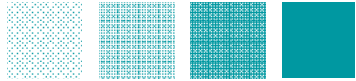


Schacht

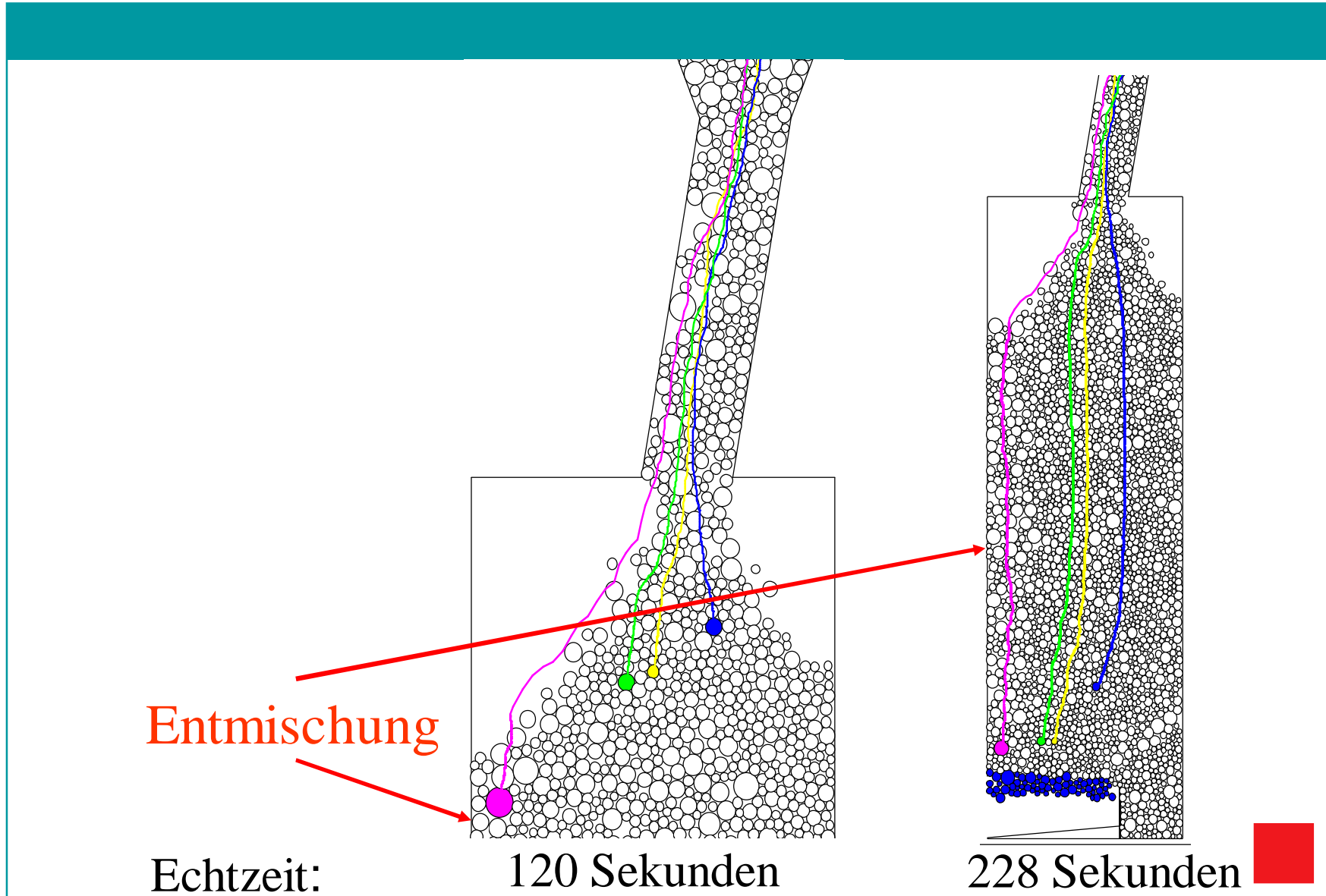


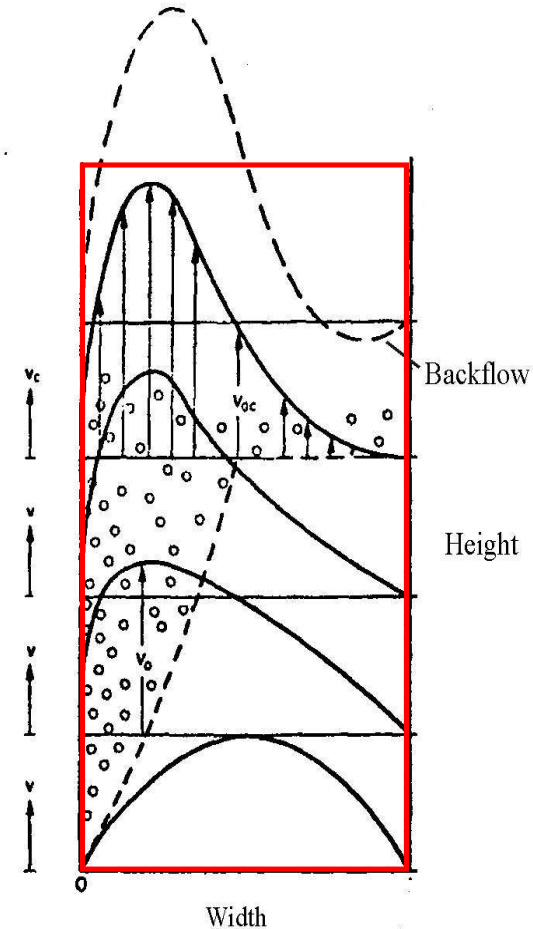
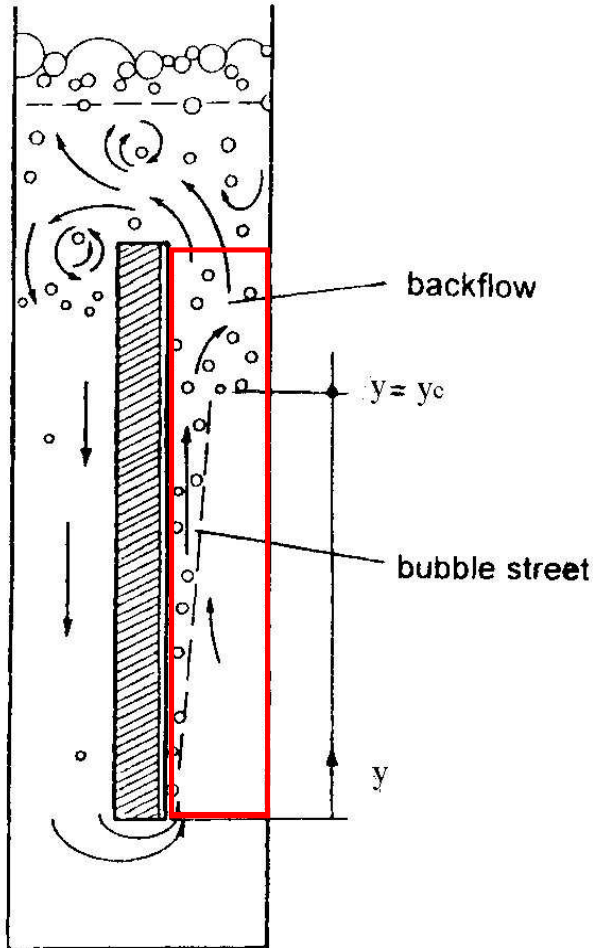
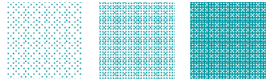
Spinnenbein



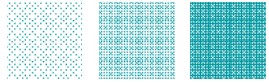


Spurverfolgung ausgewählter Partikel



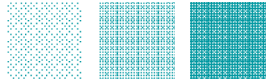


Gasentwickelnde Elektrode im elektrochemischen Reaktor

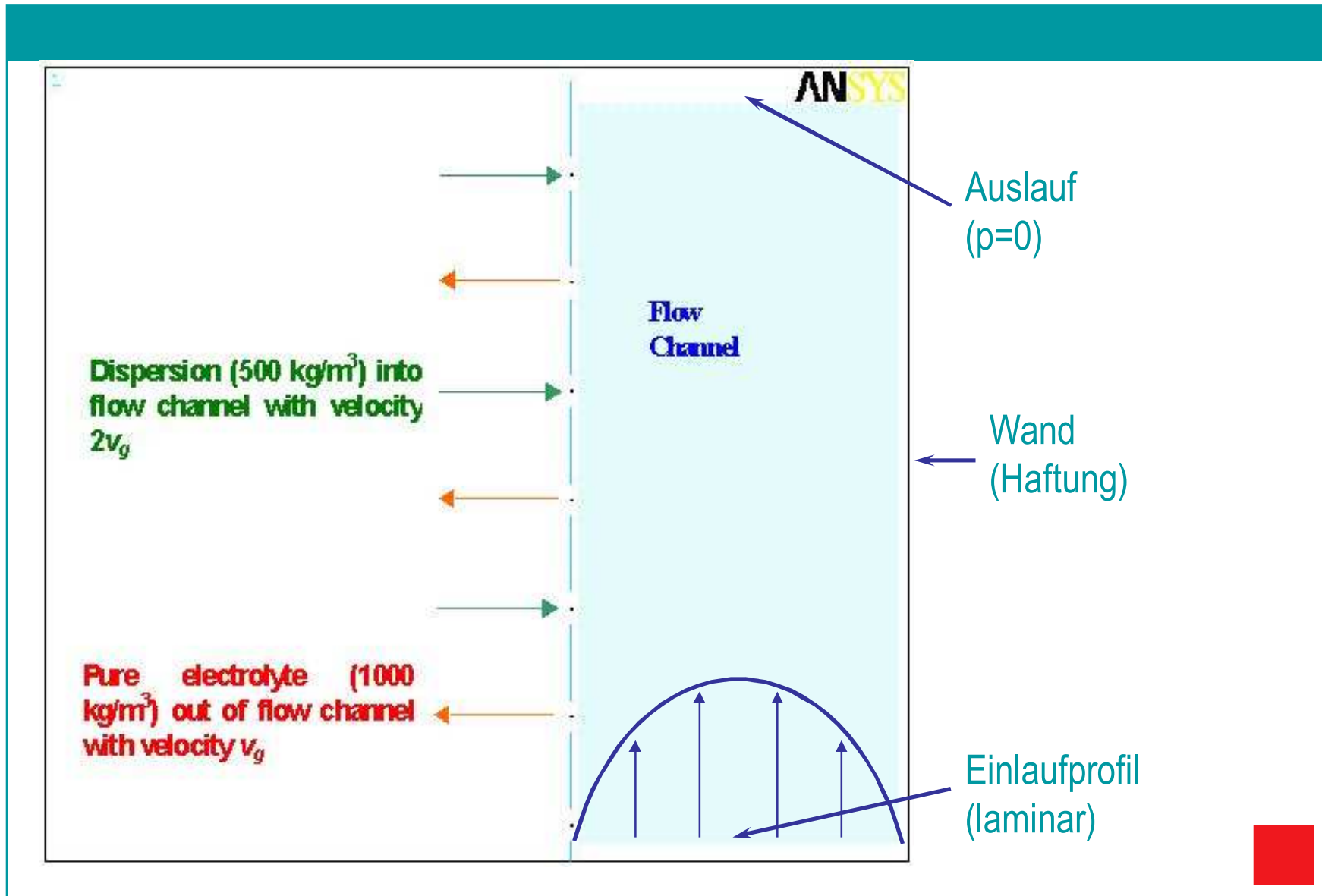


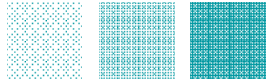
- Gasentwicklung ~ Stromstärke
- Blasendurchmesser $< 50 \mu\text{m}$
- Blasenstraße oben breiter (Schwerefeld)
- Mischdichte sinkt (fein verteilte Blasen)
- Viskosität steigt
- Rückströmung unerwünscht
- Ohmscher Widerstand steigt
- technische und wirtschaftliche Verluste



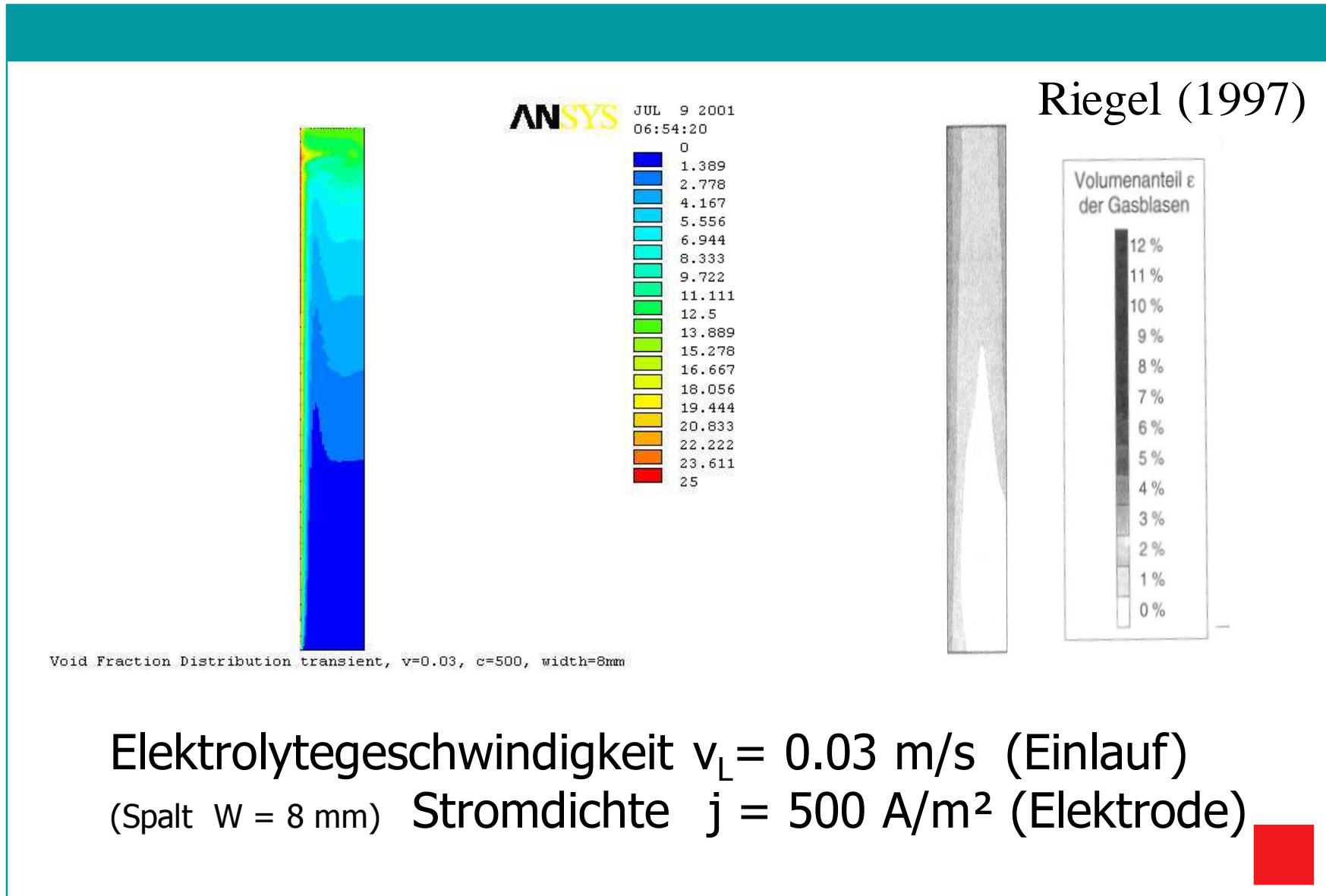


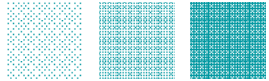
Randbedingungen



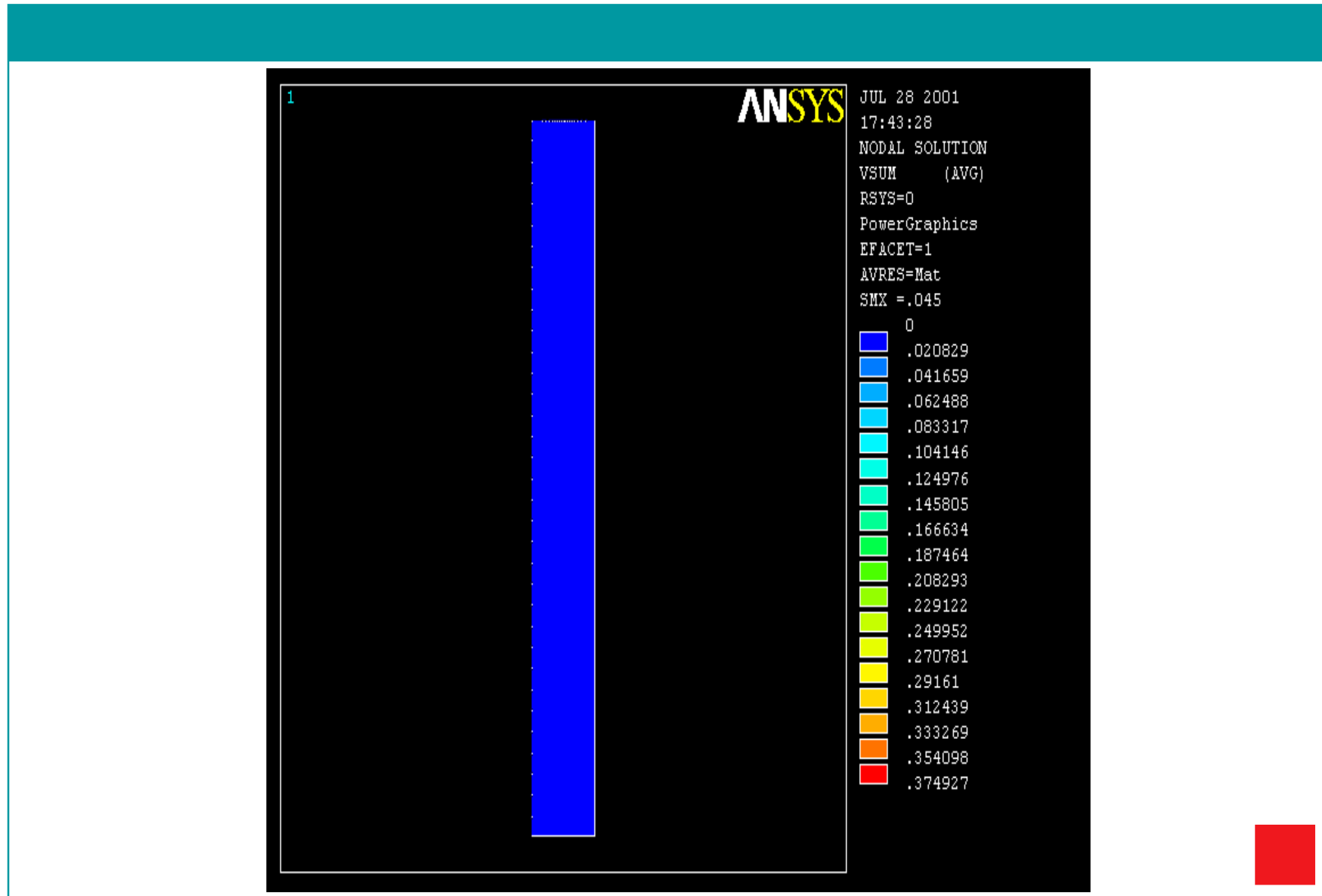


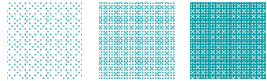
Flotranlösung versus Versuchsergebnisse



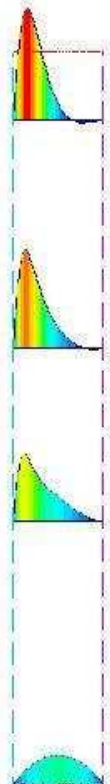


VSUM über 25 s transienter Analyse



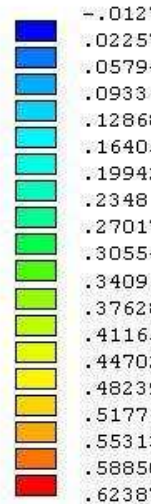


Geschwindigkeitsprofile (Spalt W = 3 mm)



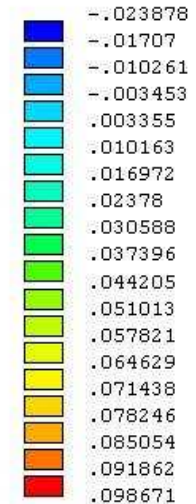
ANSYS

JUL 13 200
13:15:16
PATH= Y4
VALUE= VY
SCAL=.1



ANSYS

JUL 13 2001
13:55:57
PATH= Y4
VALUE= VY
SCAL=.05

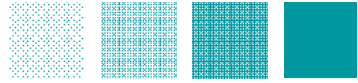


Velocity Profiles, width=3mm

$$v_L = 0.16 \text{ m/s}$$
$$j = 6250 \text{ A/m}^2$$

$$v_L = 0.026 \text{ m/s}$$
$$j = 450 \text{ A/m}^2$$





- Numerische Simulation erhöht Verständnis
- Modellwahl ist entscheidend
- Simulation spart Kosten für Versuche
- Verkürzung der Entwicklungszeit
- Nichtlineare Probleme erfordern Spezialwissen
- Erhöhte Anforderungen an die Ausbildung

Vielen Dank für Ihre Aufmerksamkeit

